Economics of Ransomware Attacks

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Abstract

Over the last few years, both the development of ransomware strains as well as changes in the marketplace for malware have greatly reduced the entry barrier for attackers to conduct large-scale ransomware attacks. In this paper, we examine how this mode of cyberattack impacts software vendors and consumer behavior. When victims face an added option to mitigate losses via a ransom payment, both the equilibrium market size and the vendor's profit under optimal pricing can actually increase in the ransom demand as well as the risk of residual losses following a ransom payment (which reflect the trustworthiness of the ransomware operator). We further show that for intermediate levels of risk of the vulnerability being successfully exploited, the vendor restricts software adoption by substantially hiking prices. This lies in stark contrast to outcomes in a benchmark case involving traditional malware (non-ransomware) where the vendor will choose to decrease price as security risk increases. Social welfare is higher under ransomware compared to the benchmark in both sufficiently low and high risk settings. However, for intermediate risk, it is better from a social standpoint if consumers do not to have an option to pay ransom. We also show that the expected total ransom paid is non-monotone in the risk of success of the attack, increasing when the risk is moderate in spite of a decreasing ransom-paying population.

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1 Introduction

In recent years, *ransomware* has evolved to become a prevalent class of malware due to improved use of encryption and attack vectors as well as increased maturity of cryptocurrency-based payment systems (such as Bitcoin and Ethereum exchanges) which obfuscate via pseudonymity the identity of transacting parties (Verizon 2018). Ransomware is an extortion-based attack that infects a computer system and subsequently prevents either access to the system (i.e., locker ransomware) or access to files or data (i.e., crypto ransomware) (Savage et al. 2015). Victims are typically threatened with permanent loss of access unless they pay a ransom. Having an additional decision for users (i.e., whether to pay ransom) disrupts the economics underlying software usage and patching behaviors, and therefore ransomware may necessitate management strategies and policies that conflict with what served prior environmental characteristics well.

While ransomware attacks often involve human interaction[1], as of late more insidious ransomware attacks that do not rely on human interaction for initial infection and/or spreading have been successfully deployed (Barkly 2017). Some of these attacks rely on identifying vulnerable systems via massive scale remote scanning of computer networks. As of 2015, a new and virulent strain has emerged - ransomware with wormlike capabilities. In this case, the malware has the ability to move *laterally* from an infected system to other unprotected systems on the same computer network without interaction or additional hacker intervention. For example, WannaCry, a ransomware worm based on leaked NSA tools, struck on May 2017 and indiscriminately affected over 230,000 computers across 150 countries in a day (Cooper 2018). One peculiar aspect of ransomware worms (compared to other ransomware strains) is that the risk of infection is characterized by *network externalities* - the more consumers that are unpatched in a network, the higher the risk for each of them.

Preventative actions are the best defense against ransomware (FBI 2016, U.S. Department of Justice 2017, No More Ransomware Project 2017). In fact, the U.S. Department of Health and Human Services delineates what healthcare providers are required to do to prevent ransomware infection in order to be HIPAA compliant (U.S. Department of Health and Human Services 2016). Timely patching of systems is considered best practices, but many organizations and users regrettably do not do so. Sadly, this state of affairs has been

[1]For example, victims' systems are compromised by opening an email attachment (typically sent via a phishing attack) or by visiting a compromised web site.

the defining characteristic of security vulnerabilities for decades, and ransomware similarly exploits the same poor vulnerability management practices.

With WannaCry ransomware, Microsoft had released a patch on March 14, 2017, following revelation of the vulnerability's existence by The Shadow Brokers hacker group (Microsoft 2017b). Two months later, despite Microsoft having made the patch available, a sizeable number of unpatched systems enabled WannaCry to spread as fast as it did. Even one month after global news outlets alerted the world to the vulnerability being exploited by WannaCry, many users and organizations had still not patched and, as a result, the NotPetya malware was able to spread using the same vulnerability (Microsoft 2017a). In fact, even a whole year after the original attack, some reputable companies had yet to complete the patching of all their systems. For example, in March 2018, several unpatched computer systems in Boeing Commercial Airplanes division were affected by WannaCry (Gates 2018). These incidents highlight how large populations of unpatched users facilitate the development and spread of ransomware, and also maintain the threat current. As software vendors and government agencies grapple with the significant losses being incurred, they have sought to understand how to respond to and operate in this new environment where consumers now face a decision of whether or not to pay ransom.

Over the past decade, ransomware experienced tremendous growth and even held the crown as the fastest growing cybersecurity threat (Cybersecurity Insiders 2017). According to Malwarebytes (2017), six out of ten malware payloads were ransomware in the first quarter of 2017. The number of ransomware variants increased 4.3 times between the first quarter of 2016 and the first quarter of 2017 (Proofpoint 2017). The number of ransomware attacks on businesses tripled in 2016, from one attack every two minutes to one attack every 40 seconds (Kaspersky 2016). Of the attacked companies, 71% had at least one machine successfully infected (Barkly 2016). Moreover, 72% of the infected businesses lost access to data for two days or more (Intermedia 2017). The overall damage that businesses incur from ransomware attacks (including remediation and lost business) is expected to reach $11.5 billion by 2019 (Morgan 2017). Because of ransomware's prevalence, businesses and users have now had experiences with the threat and importantly begun to put strategies and policies in place for managing it going forward (Ali 2016; Davis 2018; Mercer 2018). However, managing cybersecurity is a difficult task because the threat landscape can fluctuate significantly year to year driven by changing tides and the wide-ranging motivations of hackers.

Hacker motivations span human curiosity, a desire for fame, an anti-establishment agenda, economic objectives, hacktivism, and even cyberwarfare (Thomas and Stoddard 2012). Both NotPetya and WannaCry, the recent and largest ransomware attacks in history, were attributed to state actors, i.e., Russia and North Korea, respectively (Chappell and Neuman 2017; Marsh 2018). In the case of SamSam ransomware, two Iranian nationals were indicted for their involvement; these hackers seemed to be more economically motivated and earned $6 million in ransom payments (Barrett 2018). Similarly, Cryptolocker is speculated to have generated over $30 million in ransom payments in 100 days (Jeffers 2013).

In that state actors' motivations are typically political in nature, those responsible for NotPetya and WannaCry did not bother to properly set up and configure effective processes to receive payments and return decryption keys to those who paid (Greenberg 2018). Despite not having that intent, they clearly proved the feasibility of launching large-scale and disruptive ransomware attacks. In the end, only 338 WannaCry victims paid the ransom demand (Palmer 2018). An interesting question is whether these attacks could have caused greater economic damages (and been even more successful from a malicious perspective) had the ransom payment and decryption key delivery process actually been functional. For SamSam and Cryptolocker, which were clearly motivated by revenue generation, an open question is how would the scaling of such attacks to harness the worm-like characteristics of large-scale attacks impact revenues, considering that business and end users would adjust their patching and usage strategies to such threats in expectation. It is easy to see that the actual potential of ransomware has yet to be observed, and the economic models we develop in this paper aim to provide insight into what may lie on the horizon.

Most prior work has aimed at understanding how a software firm and its users react to security risk tend to model both patching costs and security losses, and these models can cover a wide variety of cyber attacks (August and Tunca 2006; Cavusoglu et al. 2008; Dey et al. 2015). However, ransomware is unique in that it presents users with an opportunity to pay in exchange for a possible reduction in security losses. Thus, we construct a model of cyber security to include these primitive elements that uniquely define ransomware. Using this model, we examine how the threat of ransomware affects consumers' choices as they face trade-offs between ex-ante security protection efforts like patching and ex-post ransom payments to agents with unlawful motives. As a class of attacks, ransomware presents a potential efficiency gain by offering a loss-mitigating payment opportunity, whereas in models of traditional attacks (see those listed above), victims typically do not have this opportunity and instead incur large valuation-dependent losses. On the other hand, such shifting of consumer incentives and their strategies modifies the network externality stemming from unpatched usage which fundamentally alters a vendor's incentives and the decision problem he faces. In totality, we seek to understand how ransomware characteristics affect software pricing, usage and security, and reflect on whether a shift in attack trends toward increased representation from the ransomware class is helpful or hurtful to the economy.

2 Literature Review

This work contributes to several research streams falling under the general topic of economics of information security, namely (i) economics of ransomware, and (ii) network security externalities due to interdependent risks. Moreover, due to the peculiarities of ransomware attacks, this work is directly related to the research stream on (iii) economic dynamics of hostage taking and negotiation.

Ransomware attacks are perpetrated based on the concept of holding hostage a digital asset and demanding a ransom for its release (Young and Yung 1996). There exists an established research stream on hostage taking, ensuing negotiations, and outcomes in scenarios involving human victims. Several empirical studies explore the effect of deterrence policies and concession making on recurrence of hijacking events (Brandt and Sandler 2009, Brandt et al. 2016) and factors impacting the attackers' perpetration and negotiation effectiveness (Gaibulloev and Sandler 2009). Other studies take a behavioral approach trying to understand terrorist actions in hostage-taking events (e.g., Wilson 2000). Early game-theoretical studies on this topic focus on the dynamics of the interaction between rational terrorists and negotiators on the part of victims (governments, families, or other interested parties). Lapan and Sandler (1988) look at multi-period scenarios where the terrorists are considering an attack each period and there are potential reputation effects propagating through time, based on government concessions during negotiations for prior attacks. They abstract the number of victims and their model characterizes attack outcomes as constants regardless of how many victims are affected. Selten (1988) explores an extension with multiple attackers and victims but each instance of an attack represents a game with an isolated outcome in which the attacker will proceed with attacking each victim separately only if he expects some benefit from the attack. Drawing parallels to cyberattacks, such modeling approaches can be used to characterize attacks that are to some extent targeted. In contrast, in the case of worm attacks (and in particular worm ransomware) executed *at scale*, even if the onset of the attack is targeted (which may not even be the case for many ransomware attacks), the brunt of the impact is due to the fact that the malware can spread laterally very fast to other unprotected systems in an untargeted way without the attacker working through a decision making process for every potential breach. In several of these attacks, the demanded ransom amount is hardcoded a priori to a default level rather than being adjusted based on the value of the compromised digital asset to the consumer (e.g., WannaCry was prompting all victims to pay \$300-\$500 per affected system). Furthermore, theoretical kidnapping models usually involve dynamics between two parties (negotiators and attackers). In contrast, many cyberattacks are enabled by vulnerabilities in an information system sold by a legitimate vendor (developer). The vendor is partially responsible for how secure his product is and can strategically create financial incentives for the adoption and patching of the system by consumers. Our framework accommodates attacks with lateral spread that are conducted at scale, and we also include the role of the vendor in influencing the size of the consumer population that is vulnerable to the attack. Beyond the existence (Young and Yung 1996) and observation of cryptovirological attacks, our work focuses on their impact on software markets and the economic incentives that govern their efficacy.

The research agenda on the economics of information security has been extensively developed along multiple directions such as patching management and incentives (Cavusoglu et al. 2008, Ioannidis et al. 2012, Dey et al. 2015, August et al. 2016, Lelarge 2009), software liability (August and Tunca 2011, Kim et al. 2011), network security (August and Tunca 2006, Chen et al. 2011, August et al. 2014), piracy (August and Tunca 2008, Lahiri 2012, Kannan et al. 2016, Dey et al. 2018, Kim et al. 2018), vulnerability disclosure (Cavusoglu and Raghunathan 2007, Arora et al. 2008, Choi et al. 2010, Mitra and Ransbotham 2015), and markets for information security and managed security services (Kannan and Telang 2005, Dey et al. 2012, Gupta and Zhdanov 2012, Ransbotham et al. 2012, Dey et al. 2014, Cezar et al. 2017). However, study of the economic dynamics of markets affected by ransomware remains relatively scarce. Different from other types of cyberattacks where the full loss is realized if the attack is successful, ransomware attacks present victims with a *post-attack* choice (and, in some cases, opportunity to negotiate): pay ransom (and hopefully retrieve access to the locked resource) or incur the full losses associated with giving up on that digital asset. From the perspective of consumers, the game is more complex. Laszka et al. (2017) explore security investments in risk mitigation (e.g, backups) and the strategic decision of whether to pay ransom. They abstract away any preventive effort investments by consumers (patching, firewalls, etc). In their study, the attacker's effort is customized to the victim, thus matching the dynamics of targeted attacks. In contrast, in our study, in the case of untargeted attacks with lateral movement, preventive actions effectively impact the spreading of the attack. Cartwright et al. (2018) adapt the models by Lapan and Sandler (1988) and Selten (1988) to ransomware attacks and explore bargaining and deterrence strategies. In particular, they show that the likelihood of irrational aggression in the absence of payment and credible commitment to return files upon receipt of payment play key roles in incentivizing victims to pay the ransom. Both of these papers consider the bargaining nature of the ransom game, where the victims have the ability to propose a counter-offer to the demanded ransom and engage in negotiations. Again, such a modeling approach is more relevant to targeted attacks on a smaller scale, where the effort is minimal on the side of the attacker to customize his handling of each victim. As mentioned above, many larger scale untargeted ransomware attacks do not allow for bargaining and the ransom is fixed into the code prior to the attack taking place. Hence, in our study, we focus more on the consumer decision of whether to pay the ransom or not in the absence of a bargaining option.

Last but not least, neither the kidnapping literature nor the extant literature on economics of ransomware capture the possibility of negative security network externalities which characterize worm cyberattacks. Cartwright et al. (2018) mention potential spillover effects of deterrence when there are two customer categories but they do not tie these effects to the size of the vulnerable population. In the case of worm ransomware, due to autonomous lateral spreading, the higher the number of unpatched systems on a network, the higher the risk of infection to every single one of them. In such attacks, a system need not be an initial target for it to eventually become compromised. Interdependent security risks have been explored in several other papers (e.g., Kunreuther and Heal 2003, Choi et al. 2010, Johnson et al. 2010, August and Tunca 2011, Hui et al. 2012, Zhao et al. 2013, Cezar et al. 2017). We extend this literature by considering risk interdependencies in the context of worm ransomware.

3 Model and Consumer Market Equilibrium

3.1 Model Description

We study the market for a software product that exhibits security vulnerabilities exploitable by worm ransomware attacks. We assume a unit-mass continuum of consumers whose valuations v for the software lie uniformly on $\mathcal{V} = [0, 1]$. When a security vulnerability arises, the vendor develops a security patch and makes it available to all users of the software. Each consumer makes a decision to buy, B, or not buy, NB. Consumers who purchase pay a price p, set by the vendor, for the product. Similarly, each buying consumer makes a decision to patch, P, or not patch, NP. Consumers who decide to patch incur an expected patching cost of $c_p > 0$. Consumers who do not patch face the risk of being hit by an attack. If unpatched, then the probability a consumer gets hit is given by πu, where $\pi > 0$ is the probability the vulnerability is exploited and u is the size of the unpatched population of users (which is endogenous to the model). With this specification, we capture the ability of a ransomware worm to autonomously replicate and spread laterally to other unpatched systems in the network.

We focus on large-scale ransomware attacks with no bargaining opportunities for the consumers. Once hit, the consumer faces two options: pay the ransom demand, R, or do nothing, NR. The representative ransomware operator demands a single ransom $R > 0$ across all victims, which is consistent with much of the ransomware in this family (Symantec 2016, F-Secure 2016). A ransomware victim of type v that does not pay the ransom incurs losses αv, where $\alpha > 0$. On the other hand, even consumers who pay ransom face a risk that the attacker may not release the decryption key. This can happen for multiple reasons that interact with the wide-ranging attacker motivations discussed in Section 1. For example, an economically-motivated attacker may not release the key because he aims to extract more out of the victims (Siwicki 2016). Or perhaps not releasing decryption keys is a result of unintentional failures in either a manual process for producing and releasing keys or in the systems that process ransom payments (Abrams 2016). Attackers with either political motivations or other motivations less economic in nature may not have any intention to produce or release the keys in the first place (Frenkel et al. 2017, Marsh 2018).

Because users face uncertain ransomware risks based on uncertain motivations, we parameterize the primary loss characteristics faced by users which presents the ability to analyze outcomes *across* the varied motivations that underlie hacker activity. In particular, users who pay ransom still incur some residual valuation-dependent losses in expectation. We model them as scaled losses by a factor $\delta \in [0, 1]$. For example, $\delta = 1$ represents the case where the ransomware operator has no intention of releasing the decryption keys upon payment, and a smaller δ represents the opposite case where the operator uses well-functioning, automated decryption key release systems and residual losses to paying users are minimal.

In general, one can vary over the (R, δ) parameter space to map to hacker motivations and then gain insights into the equilibria that unfold when ransomware has characteristics consistent with each motivation. This parametric approach is preferable here because the wide-ranging and disparate motivations behind observed ransomware would make objective specification (in malicious agent modeling) untenable. Moreover, it permits broader insights into a threat landscape that is quite dynamic in nature; the version and intent of ransomware seen today in WannaCry and NotPetya may look starkly different than the successful ransomware campaign of tomorrow which our model intends to inform upon.

The consumer action space is $S = \{(B, P), (B, NP, R), (B, NP, NR), (NB)\}$ and for a given strategy profile $\sigma : \mathcal{V} \to S$, the expected utility function for consumer v is given by:

$$U_{RW}(v, \sigma) \triangleq \begin{cases} v - p - c_p & if \quad \sigma(v) = (B, P)\,; \\ v - p - \pi u(\sigma)(R + \delta\alpha v) & if \quad \sigma(v) = (B, NP, R)\,; \\ v - p - \pi\alpha u(\sigma) v & if \quad \sigma(v) = (B, NP, NR)\,; \\ 0 & if \quad \sigma(v) = (NB)\,, \end{cases} \tag{1}$$

where $u(\sigma) \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma(v) \in \{(B,NP,R),(B,NP,NR)\}\}}\, dv$ is the size of the unpatched adopting population in the presence of the ransomware threat. Without loss of generality, we assume that $\delta \in [0, 1]$, $\pi \in (0, 1)$, $c_p \in (0, 1)$, $R \in [0, \infty)$, and $\alpha \in (0, \infty)$.

Beyond developing an understanding of market dynamics in the presence of ransomware threats, one of our goals is to highlight differences in comparison to scenarios involving traditional malware threats where victims do not have the option to pay ransom to mitigate the impact of the attack. For this purpose, we use as a comparison *benchmark* the model from August and Tunca (2006), which captures fundamental characteristics of software markets in the presence of malware attacks with no ransom option. Their model considers only three potential consumer strategies, $S_{BM} = \{(B, P), (B, NP), (NB, NP)\}$, but still captures the network externalities. For a given strategy profile $\sigma : \mathcal{V} \to S$, the expected utility function for consumer v is given by:

$$U_{BM}(v,\sigma) \triangleq \begin{cases} v - p - c_p & if \quad \sigma(v) = (B,P)\,; \\ v - p - \pi\alpha u_{BM}(\sigma)v & if \quad \sigma(v) = (B,NP)\,; \\ 0 & if \quad \sigma(v) = (NB)\,, \end{cases} \tag{2}$$

where $u_{BM}(\sigma) \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma(v)\in\{(B,NP)\}\}}\,dv$ is the size of the unpatched adopting population under the benchmark case.

3.2 Consumer Market Equilibrium

Before examining the impact of ransomware on the vendor's decision, we first must characterize how consumers behave in equilibrium for a given price. There are two factors that complicate their decisions. First, the level of risk upon being unpatched is endogenously determined by the actions of consumers. Second, this risk includes the behavior of both those who would pay ransom as well as those who would not. Thus, we first focus on understanding the effect of their strategic interactions on equilibrium behavior due to the externality generated by both subpopulations. The consumer with valuation v selects an action that solves the following maximization problem: $\max_{s\in S} \; U_{RW}(v,\sigma)\,,$ where the strategy profile σ is composed of σ_{-v} (which is taken as fixed) and the choice being made, i.e., $\sigma(v) = s$. We denote the optimal action that solves her problem with $s^*(v)$. Further, we denote the equilibrium strategy profile with σ^*, and it satisfies the requirement that $\sigma^*(v) = s^*(v)$ for all $v \in \mathcal{V}$. We next characterize the structure of the consumer market equilibrium that arises.

Lemma 1. *Given a price p and a set of parameters $\pi, \alpha, c_p, R,$ and δ, there exists a unique equilibrium consumer strategy profile σ^* that is characterized by thresholds v_{nr}, v_r, $v_p \in [0,1]$. For each $v \in \mathcal{V}$, it satisfies*

$$\sigma^*(v) = \begin{cases} (B,P) & if \quad v_p < v \le 1\,; \\ (B,NP,R) & if \quad v_r < v \le v_p\,; \\ (B,NP,NR) & if \quad v_{nr} < v \le v_r\,; \\ (NB) & if \quad 0 \le v \le v_{nr}\,. \end{cases} \tag{3}$$

Lemma 1 establishes that the consumer market equilibrium has a threshold structure. The highest-valuation consumers have the most value to lose if attacked, so they patch in equilibrium (if the risk is sufficiently high). Those with lower valuations remain unpatched, trading off a fixed cost of patching for valuation-dependent losses. Of those who are unpatched, those with higher valuations are the ones who pay ransom to reduce the impact of being unpatched on their valuation-dependent losses. Importantly, it can be the case that no unpatched consumer (if R or δ is sufficiently high) pays ransom or even all unpatched consumers pay ransom (if R or δ is sufficiently low).

We denote the vendor's profit function by $\Pi(p) = p \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v|p) \in \{(B,NP,NR),(B,NP,R)(B,P)\}\}} dv$, noting that marginal costs are assumed to be negligible for information goods. The vendor sets a price p for the software by solving the following problem: $\max_{p \in [0,\infty)} \Pi(p)$, such that (v_{nr}, v_r, v_p) are given by $\sigma^*(\cdot \,|\, p)$. With the optimal price p^* that solves the vendor's problem, we denote the profits associated with this optimal price by $\Pi^* \triangleq \Pi(p^*)$. In the next section, we will discuss how R and δ play a role in impacting the vendor's pricing strategy and, ultimately, in shaping equilibrium consumer behavior.

4 Impact of Ransomware Characteristics

4.1 Pricing-induced Market Structures

Because our model parameter space induces many different equilibria including those not commonly found in practical settings, it is worthwhile to focus our analysis on subspaces that are more relevant. As a simple example, it is natural that, in equilibrium, if patching costs (c_p) are really low then most users patch and if patching costs are too high, then no user patches; however, neither outcome is characteristic of settings that are commonly observed. To better focus on regions where key trade-offs are more active, we make the following assumptions going forward:

Assumption 1. $\frac{1}{2}\left(3 - 2\sqrt{2}\right) < c_p < \frac{1}{2}\left(2 - \sqrt{2}\right)$, *and*

Assumption 2. $\underline{\alpha} < \alpha < \overline{\alpha}$,

where $\underline{\alpha} = \frac{c_p(2-c_p)^2}{(1-c_p)^2}$ and $\overline{\alpha} = \frac{-3+\sqrt{(1-c_p)(9-25c_p)}+c_p(46-75c_p+15\sqrt{(1-c_p)(9-25c_p)})}{16(1-3c_p)}$.

While costs of patching involve inefficiencies due to downtime during the patching process, usually the patch distribution and installation processes have been greatly streamlined and sometimes automated for individual consumer and enterprise systems. When patching is done properly and in tandem with the adoption of fail-safe measures (such as restoration capabilities / backups) the associated business costs are usually within reasonable ranges. Similarly, such fail-safe measures can reduce the extent of damage of a ransomware (or other type of malware) attack. For simplicity, in our model, we assume that patching is effective at preventing the exploitation of the vulnerability. Moreover, the assumptions on c_p and α above are sufficient conditions to obtain the findings in our paper, which can extend well beyond this focal region. Also note that our model can capture scenarios in which consumers' losses if hit can exceed their valuations of the software ($\alpha > 1$).[2]

It will be helpful to better understand how the vendor changes the consumer market structure he induces via pricing in equilibrium based on some characteristics of the ransomware setting, e.g. size of the ransom demand R and security risk factor level π. Figure 1 provides a helpful illustration of how the consumer market equilibrium outcome fluctuates. This figure depicts an instance of the focal region we study (defined by the assumptions above), and it will be useful as a reference for the reader in keeping in mind which structures are in play. Moreover, some subsequent figures study vertical and horizontal slices across Figure 1 which can more easily be visualized here. For consistency, the capital letter labels in these cases also refer back to the region labels of Figure 1.

Figure 1 shows that when R and π are sufficiently low, then prices are set such that all customers of the software opt to remain unpatched and pay the ransom if hit. Overall, the expected losses are sufficiently low in Region (A) of the figure that consumers do not find it worthwhile to incur the cost to protect themselves by patching, and, if hit, they also prefer to pay the ransom because R is relatively low. On the other extreme, if R and π are sufficiently high, then the equilibrium outcome would be identical to the outcome observed in a world without ransom. This is seen in Region (D), in which the equilibrium outcome is $0 < v_{nr} < v_p < 1$. In a setting with a high security risk factor π, higher-valuation consumers have a strong incentive to protect themselves from risk by patching. Those with lower valuations prefer to remain unpatched and do not pay high ransom demands; this results in the market outcome described. In the middle ground between these two scenarios, we see that the equilibrium outcome aligns well with observations of the world

[2]For example, losses associated with reputation, customer goodwill and attack clean-up can often be substantial and exceed a user's valuation of the software itself. However, *expected* losses are necessarily less than the user's valuation. The user only makes trade-offs between paying ransom and incurring losses at the last stage of the game, consistent with the sub-game perfect equilibrium solution concept we employ.

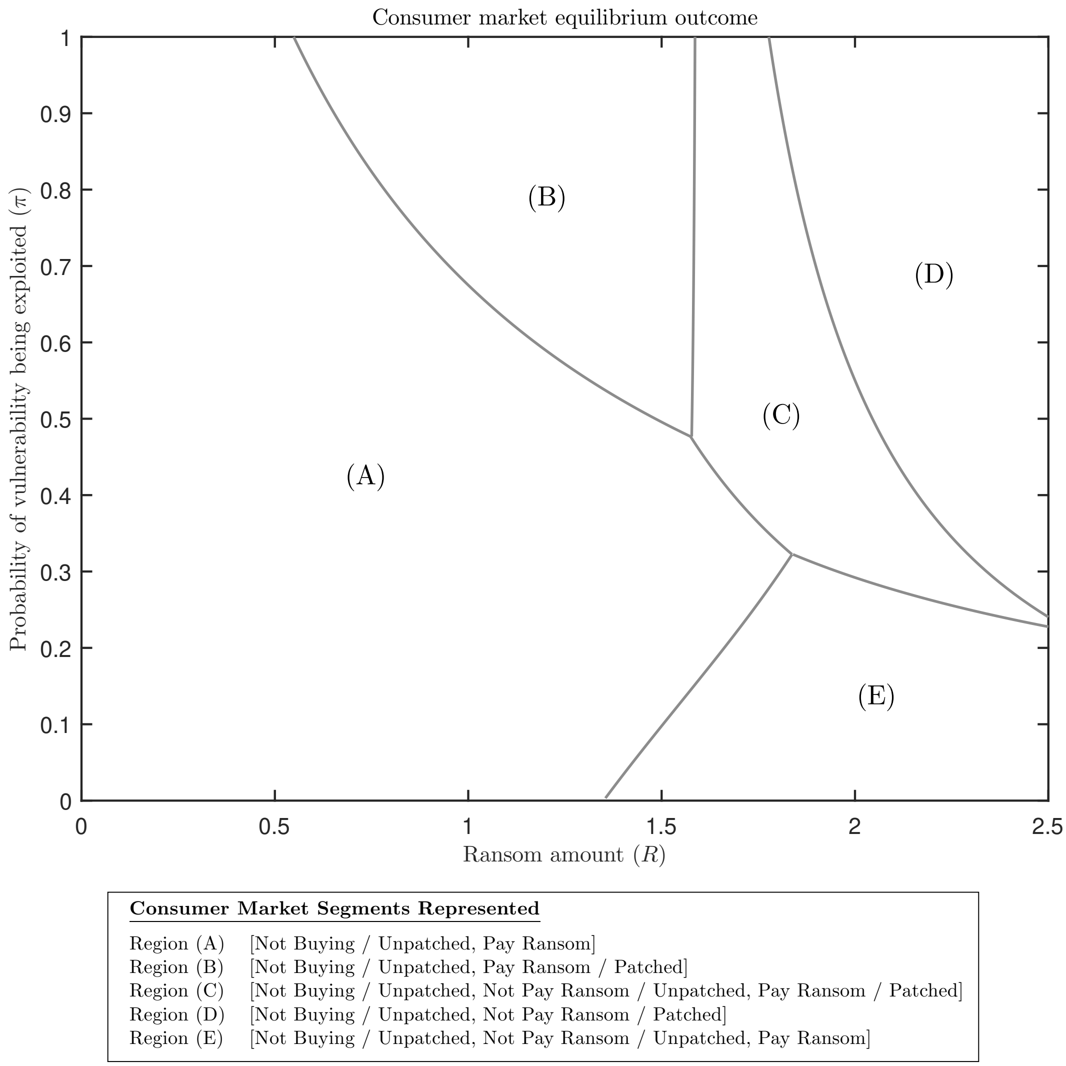

Figure 1: Characterization of equilibrium consumer market structures across regions in the ransom demanded (R) and security loss factor (π). Region labels describe the consumer segments that arise in each region in order of increasing consumer valuations (from left to right). Patching costs ($c_p = 0.28$), security loss factor ($\alpha = 3$), and residual loss factor ($\delta = 0.1$) are selected to ensure all consumer patching and ransom paying behaviors are present for some sub-region.

today. In Region (C), the consumer market outcome is characterized by $0 < v_{nr} < v_r < v_p < 1$, in which some of the customers who opt to remain unpatched choose to pay the ransom if hit. Region (C) will be in the spotlight throughout the paper, as it represents a region of the parameter space corresponding well to practical outcomes. For completeness, we will also examine neighboring parameter regions to portray a sense of how the equilibrium outcome unfolds with perturbations in the levels of risk. In particular, if the risk factor is not as high as in Region (C), then one might expect $0 < v_{nr} < v_r < 1$ to arise in equilibrium, in which no customer patches and higher-valuation customers opt to pay ransom if hit. Such a region does arise, and it is depicted as Region (E). Similarly, with a high risk factor but a smaller expected ransom demand, the outcome $0 < v_r < v_p < 1$ in which the highest-valuation customers patch and all unpatched customers pay the ransom if hit also arises, depicted as Region (B).

4.2 Role of the Ransom Amount, Risk and Residual Losses

With the newfound understanding of how the equilibrium outcome unfolds across different regions of the parameter space, we next investigate regions of interest in more depth. In the rest of this section, we describe and illustrate several insights into ransomware economics. For example, one might expect that a higher ransom demand would negatively impact the vendor and reduce the market share of the affected product. However, that is not always the case, as is shown in the first proposition. For the majority of results and discussions in this paper, we focus on residual losses for ransom-paying consumers being low (i.e., δ satisfying an upper bound). This assumption matches more recent ransomware trends (Disparte 2018). An economically-motivated hacker would generally deploy ransomware with characteristics satisfying such conditions because the hacker's goal is to generate ransom payments which would be negatively impacted by post-payment malicious behavior. Despite ransom-paying consumers requiring some belief about "honor among thieves", for certain classes of hacker motivations maintaining this honor would be in everyone's best interest (Fleishman 2016). To gain a broader view into diverse motivations and provide an overall more comprehensive analysis, we relax this assumption in Proposition 5 and again in Section 4.3.2 to discuss scenarios of high residual losses.

Proposition 1. *There exists a bound* $\tilde{\delta} > 0$ *such that if* $\delta < \tilde{\delta}$*:*

(a) if $c_p\alpha < R < \frac{\alpha}{2-c_p}$ *and* $\frac{c_p\alpha}{R^2 - c_p R\alpha} < \pi \leq 1$*, then the equilibrium consumer market structure is* $0 < v_r < v_p < 1$ *and the vendor's profit under equilibrium pricing decreases in* R*. Furthermore, the size of the market decreases in* R *despite the vendor's optimal price also decreasing in* R*.*

(b) if $\frac{\alpha}{2-c_p} < R < \tilde{R}$ *and* $\frac{c_p\alpha}{R^2 - c_p R\alpha} < \pi \leq 1$*, then the equilibrium consumer market structure is* $0 < v_{nr} < v_r < v_p < 1$ *and the vendor's profit under equilibrium pricing increases in* R*.*

Moreover, the size of the market increases in R even though the vendor's optimal price also increases in R.[3]

When the ransom demand is not too low and the potential losses from the attack are sufficiently high regardless of whether the victim pays ransom or not, then high-valuation consumers elect to patch. It is important to note that the trade-off here centers on c_p versus $\pi u(\sigma)(R + \delta\alpha v)$ (i.e., the *expected* costs under a ransom-paying strategy). Therefore, a patching population only emerges when R is large relative to c_p, in that the likelihood of an attack striking, $\pi u(\sigma)$, can be low in equilibrium. Part (a) of Proposition 1 pertains to a region of the parameter space in which the ransom demand is low enough that all unpatched consumers simply pay the ransom if hit (but not so low that nobody patches). This corresponds to Region (B) of Figure 1. Since all unpatched customers pay ransom, all of them are negatively impacted by an increase in R. In particular, given some ransom demand R, the consumer of type v_r (the consumer indifferent between not purchasing and purchasing) derives zero surplus upon purchasing the software. An increase in R induces this customer (and others with low valuations) to strictly prefer not purchasing at all. In that way, an increase in R hurts the vendor when all unpatched customers are paying the ransom.

As R moves relatively higher, the unpatched population splits into subpopulations of ransom payers and non-payers. This corresponds to Region (C) of Figure 1. An increase in R incentivizes some unpatched consumers who would have paid ransom to strictly prefer patching over remaining unpatched and risk getting hit with ransomware. This, in turn, leads to a reduction in the size of the unpatched segment, thus reducing the risk of an attack. As a result, those on the bottom end of the market who were unpatched but not paying ransom now have higher surplus upon remaining unpatched. Consequently, consumers of even lower valuations (who would not be in the market under a smaller R) now find it incentive-compatible to use the software and remain unpatched, and the vendor is able to profitably extract greater surplus by charging a higher price.

Figure 2 illustrates how an increase in the ransom demand (R) can benefit the vendor, with the market size expanding despite an increase in price. In particular, panel (a) depicts how the equilibrium consumer market structure changes in the ransom demand. Panel (a)

[3]For convenience, we define $\tilde{R} = \min\left(\frac{1}{2}\alpha(1 + c_p), \frac{1}{4}\left(\alpha + \sqrt{\alpha(16c_p + \alpha)}\right)\right)$.

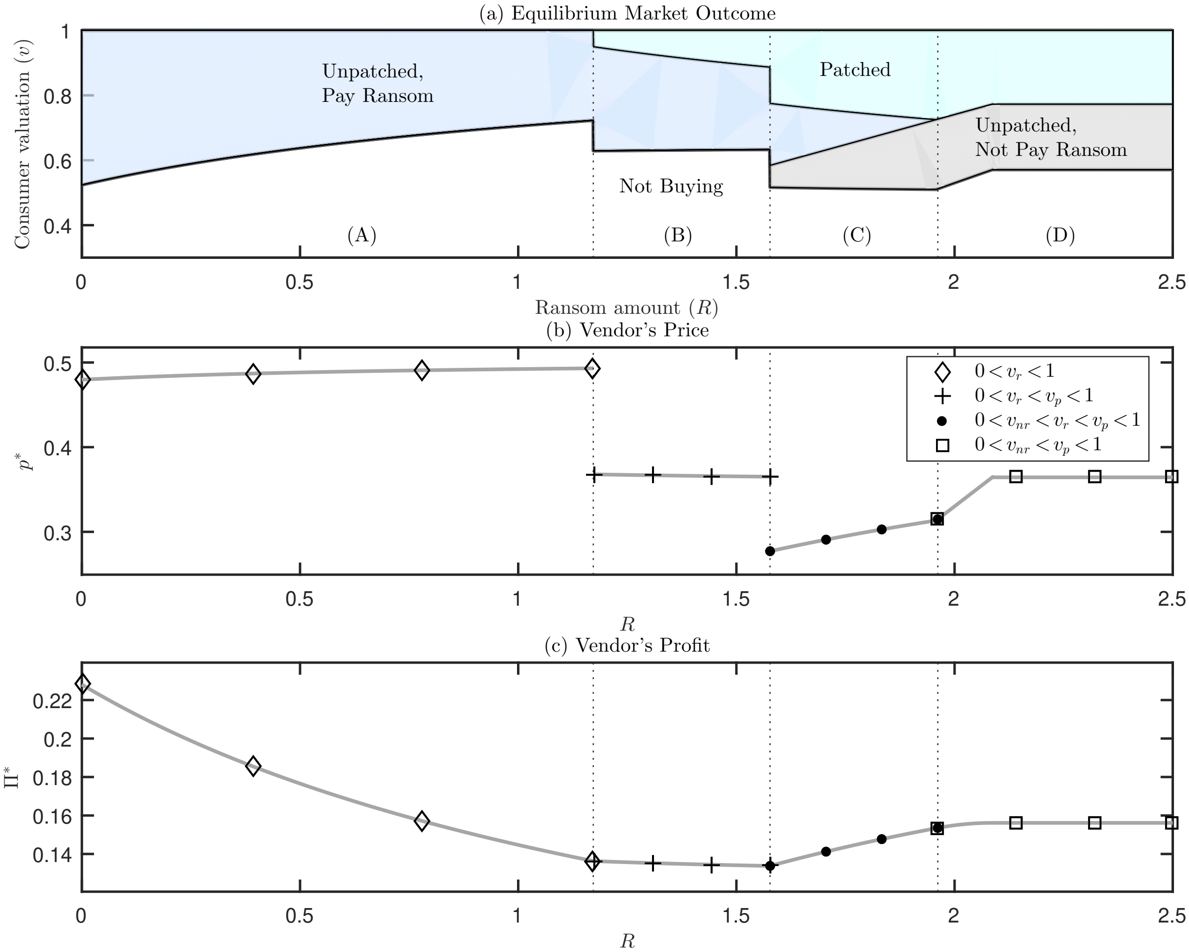

Figure 2: Impact of ransom demand (R) on the equilibrium market outcome, vendor's price, and the vendor's profit. The parameter values are $c_p = 0.28$, $\alpha = 3$, $\delta = 0.1$, and $\pi = 0.6$. The capitalized letter region labels correspond to region labels in Figure 1. The legend in panel (b) also applies to panel (c).

illustrates that when the ransom demand is high enough (roughly $R > 1.96$ in the figure), then no unpatched customer pays ransom if hit. For a slightly lower ransom demand (roughly between $R = 1.58$ and $R = 1.96$), then higher-valuation unpatched consumers pay ransom if hit while those of lower valuations do not. In particular, in panel (a) we see the market structure $0 < v_{nr} < v_r < v_p < 1$ arises in equilibrium under vendor pricing for this range of R. This market structure (in which all segments of the market are present) is the equilibrium market structure for which the insights of part (b) of Proposition 1 are applicable.

The direct impact of an increase in the ransom demand on consumer behavior is illustrated in panel (a). Between $R = 1.58$ and $R = 1.96$, the threshold valuation (marking the boundary between the patched population and the unpatched/ransom-paying population) decreases in R. An increase in the ransom demand causes those indifferent between patching and not patching to strictly favor patching, precisely because, in this range of R, higher-valuation unpatched consumers will pay ransom demands if hit. On the other hand, unpatched consumers with lower valuations are not directly impacted by an increase in R, because they do not pay ransom anyway. However, these consumers are indirectly impacted by R; particularly, the negative externality that they endure upon remaining unpatched is reduced due to increased patching behavior from higher-valuation consumers. As a result, all else being equal these unpatched customers are now better off when observing a higher ransom demand being charged, and the market size thus expands in R. This behavior can be seen in panel (a), as the lower-most threshold valuation (i.e., the boundary between the unpatched/non-paying population) decreases in R.

As a result of this dynamic, the vendor is also now better off with a higher R. Even holding price fixed, more customers adopt as R increases. What is compelling is that an increase in ransom demand provides natural incentives to patch in a way that only negatively affects a sub-segment of the unpatched population. Only those who pay ransoms are negatively impacted while those who do not actually benefit with an increase in the severity of the extortion. Consequently, the vendor has incentives to increase the price of the software as the ransom demand increases, which can be seen in panel (b), to extract back some surplus from those lower-valuation unpatched consumers who benefitted. Ultimately, the vendor benefits from an increase in the ransom demand when all market segments are present, as is illustrated in panel (c) between $R = 1.58$ and $R = 1.96$.

In contrast, the dynamic discussed above is absent when the ransom demand is small enough that all unpatched consumers simply pay ransom if hit. Referring back to Figure 2, when $R < 1.58$, the equilibrium market structure is given by $0 < v_r < v_p < 1$ (which corresponds to part (a) of Proposition 1). Therefore, in equilibrium, all unpatched consumers are directly and negatively impacted by an increase in R. In this case, those lower-valuation consumers now also feel the pinch associated with an increase in R, which drives the consumer indifferent between not purchasing and purchasing but remaining unpatched to now strictly prefer to exit the market. Contrasting with the above region, rather than benefitting the vendor, an increase in R ultimately hurts the vendor.

The discussion above demonstrates how ransomware is fundamentally different from other forms of attack in which victims have little recourse. By offering victims a chance to reduce their losses, ransomware attackers can sometimes segment the unpatched user population into two interdependent tiers. The expansion or reduction of either tier indirectly impacts both tiers simultaneously because all unpatched hosts are potential vectors for the spread of ransomware. But now, in contrast to traditional modes of attack, an increase in the ransom demand may *directly* affect only a single tier which helps the vendor to discriminate.

Besides understanding how the market is expected to evolve as attackers demand higher ransoms, we also want to explore how the market changes as the inherent risk (π) of the software being breached increases. We first analyze how the vendor adjusts his pricing strategy with respect to risk.

Proposition 2. *There exist bounds* $\tilde{\delta}, \underline{\pi}_L, \bar{\pi}_L, \underline{\pi}_M, \bar{\pi}_M, \underline{\pi}_H > 0$ *satisfying the conditions*

$$\underline{\pi}_L < \bar{\pi}_L < \frac{\alpha - 2R}{3R^2 - 4\alpha R + \alpha^2} < \underline{\pi}_M < \bar{\pi}_M < \frac{c_p \alpha}{R^2 - c_p R \alpha} < \underline{\pi}_H$$

such that if $\delta < \tilde{\delta}$ *and* $\frac{\alpha}{2-c_p} < R < \tilde{R}$*, the vendor's equilibrium price* p^* *is decreasing in* π *on* $(\underline{\pi}_L, \bar{\pi}_L)$*,* $(\underline{\pi}_M, \bar{\pi}_M)$ *and* $(\underline{\pi}_H, 1)$*. But,*

$$p^*|_{\pi \in (\underline{\pi}_M, \bar{\pi}_M)} > \max\left(p^*|_{\pi \in (\underline{\pi}_L, \bar{\pi}_L)}, p^*|_{\pi \in (\underline{\pi}_H, 1)}\right).$$

When the inherent risk factor is low, i.e., $\pi \in (\underline{\pi}_L, \bar{\pi}_L)$, consumers have no reason to patch. However, for a higher range of ransom demands as specified in Proposition 2, consumers split in their decision to pay ransom. In particular, the unpatched population separates into two sub-populations in terms of their equilibrium strategies (those who pay and those who do not pay if hit) leading to a consumer market outcome characterized by $0 < v_{nr} < v_r < 1$). As the inherent risk factor increases through this range, the vendor mitigates the impact of increased risk on his customers by lowering price. However, when the risk factor increases further to $\pi \in (\underline{\pi}_M, \bar{\pi}_M)$, there is a significant and strategic change in the equilibrium pricing behavior of the vendor. This is the primary message of Proposition 2: when the risk factor is in a middle range, the vendor strategically increases price to focus only on higher-valuation customers. This pricing strategy changes the consumer market characterization to $0 < v_r < 1$ in which all purchasing consumer remain unpatched and pay ransoms if hit. Lower-valuation customers drop out of the market due to the increased risk and higher price. Notably, as we will see later in Section 4.3, such a strategic increase in pricing can never happen in the benchmark case when ransomware is not present in the market.

One question of interest when examining the impact of the inherent risk factor on a software market is how the expected total ransom paid by victims gets affected. Although the size of the consumer population willing to pay ransom if hit (r) always shrinks as this risk factor increases (as seen in panel (c) of Figure 3), it turns out that the expected total ransom paid is in fact non-monotone which is illustrated in panel (e) of Figure 3.

Proposition 3. *There exist bounds $\tilde{\delta}, \underline{\underline{\pi}}_M, \bar{\bar{\pi}}_M, \underline{\underline{\pi}}_H > 0$ satisfying the condition*

$$\frac{\alpha - 2R}{3R^2 - 4\alpha R + \alpha^2} < \underline{\underline{\pi}}_M < \bar{\bar{\pi}}_M < \frac{c_p \alpha}{R^2 - c_p R \alpha} < \underline{\underline{\pi}}_H < 1$$

such that if $\delta < \tilde{\delta}$ and $\frac{\alpha}{2-c_p} < R < \tilde{R}$, the size of the consumer population whose strategy is to pay ransom if hit, $r(\sigma^*)$, decreases in π for all $\pi > \frac{\alpha-2R}{3R^2-4\alpha R+\alpha^2}$. However, the expected total ransom paid increases in π on $(\underline{\underline{\pi}}_M, \bar{\bar{\pi}}_M)$ and decreases in π on $(\underline{\underline{\pi}}_H, 1)$.

Figure 3 illustrates Proposition 3. In particular, panel (c) plots the size of the consumer segment that pays ransom if hit, $r(\sigma^*)$, i.e., the mass of consumers whose equilibrium strategy is (B, NP, R). Panel (d) illustrates the total mass of consumers who remain unpatched in equilibrium, $u(\sigma^*)$, i.e., the mass of consumers choosing either (B, NP, NR) or (B, NP, R), while panel (e) illustrates the expected total ransom paid, i.e., $T(\sigma^*) \triangleq \pi u(\sigma^*) r(\sigma^*) R$. The key point to note here is that the externality $u(\sigma^*)$ depends not only on those consumers willing to pay the ransom but also on those who remain unpatched and are not willing to pay. Referencing the left-hand side of panel (a) in Figure 3, since the ransom demand R is moderate, the unpatched population splits into the two tiers when the inherent risk π is low (resulting in a market structure $0 < v_{nr} < v_r < 1$). Note that the consumer indifferent between paying and not paying does not depend on the risk, since at the decision node at which a consumer is making a decision of whether or not to pay, the tradeoff is between incurring a loss of αv and incurring a loss of $R + \delta\alpha v$. The valuation of the consumer indifferent between paying ransom and remaining unpatched versus just staying unpatched is given by $v = \frac{R}{\alpha(1-\delta)}$. Since consumers who remain unpatched but do not pay ransom have valuations even lower than this threshold, they are the ones to first drop out of the market as π increases. Consequently, the size of the consumer population willing to pay ransom if hit remains constant in π at first (see the left-hand side of panel (c)). Although $u(\sigma^*)$

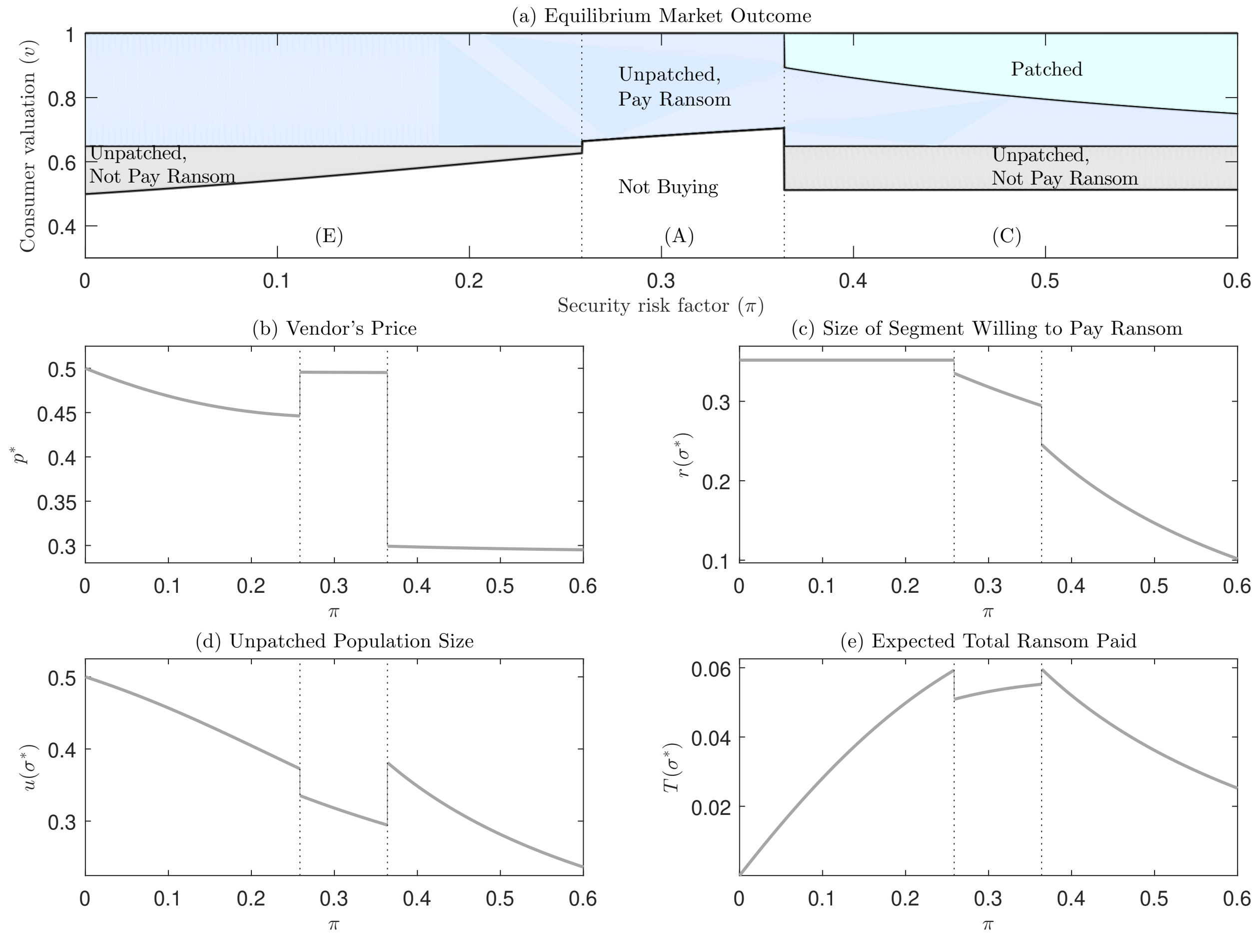

Figure 3: Impact of risk factor (π) on the equilibrium market outcome, vendor's price, size of the market segment willing to pay ransom if hit, size of the market segment willing to remain unpatched, and expected total ransom paid. The parameter values are $c_p = 0.28$, $\alpha = 3$, $\delta = 0.1$, and $R = 1.75$.

necessarily shrinks, the overall risk $\pi u(\sigma^*)$ increases as π increases such that the expected total paid by victims is also increasing in π (illustrated in panel (e), to the left of $\pi = 0.26$).

When the vendor strategically increases price (for π between 0.26 and 0.36) as discussed in Proposition 2, those low-valuation consumers who had been choosing (B, NR, NP) drop out of the market and also the size of the ransom-paying group shrinks which is now the only segment present in the market. In the immediate vicinity of $\pi = 0.26$, as the market structure changes, the overall risk externality $\pi u(\sigma^*)$ (with $u(\sigma^*) = r(\sigma^*)$) suddenly drops. The net impact of these two effects is a drop in the size of the expected total ransom paid by victims around $\pi = 0.26$. Nevertheless, as risk increases, the ransom-paying population does not decrease steeply. Hence, in this risk range, the expected total ransom paid by victims remains monotone increasing in π, albeit trailing behind the levels just prior to the market structure change.

Around $\pi = 0.36$, the vendor finds it optimal to significantly drop price and all three market segments emerge in equilibrium (i.e., $0 < v_{nr} < v_r < v_p < 1$). The significant drop in price invites additional consumers to enter at the low end of the market and stay unpatched. The increased risk associated with that entry induces the high valuation customers to shield themselves from the risk by patching. In addition, the ransom-paying population keeps shrinking. The sudden jump in the unpatched population $u(\sigma^*)$ compensates for the drop in $r(\sigma^*)$, and overall the total expected ransom paid momentarily jumps upward. But perhaps what is more notable is the change in monotonicity. As the risk increases even further, higher-valuation consumers who were willing to just pay ransom if hit switch to patching at such a rate that those low-valuation consumers who do not pay ransom are only marginally impacted by the increased risk. Moreover, as discussed before, the marginal customer indifferent between paying and not paying ransom is not affected by the overall risk level. Thus, both the ransom-paying population $r(\sigma^*)$ and the overall unpatched population $u(\sigma^*)$ keep shrinking. Nevertheless, unlike the low risk region, this dual shrinking effect dominates the increase in the risk factor (π) and the expected overall ransom paid decreases.

While the option to pay ransom offers a recourse to mitigate value-dependent losses, it also involves a secondary risk. When considering the ransom payment, victims in general are not sure a priori that the attacker will deliver the promised decryption keys. As mentioned in Section 3, this secondary risk is captured by the parameter δ. In the remainder of this section, we explore how δ impacts the vendor's profit, expected aggregate losses incurred by the unpatched population, and aggregate consumer surplus. The latter two measures are defined by:

$$UL \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,NR)\}} \pi\alpha u(\sigma^*) v dv + \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,R)\}} \pi u(\sigma^*)(R + \delta\alpha v) dv \,,$$

$$CS \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)\in\{(B,NP,NR),(B,NP,R),(B,P)\}\}} U_{RW}(v, \sigma^*) dv.$$

Proposition 4. *There exists a bound $\tilde{\delta} > 0$ such that if $\delta < \tilde{\delta}$, $\frac{\alpha}{2-c_p} < R < \tilde{R}$, and $\frac{c_p\alpha}{R^2 - c_p R\alpha} < \pi \leq 1$, then:*

(a) the vendor's profit will increase in δ, and

(b) the aggregate unpatched losses and consumer surplus both decrease in δ.

Proposition 4 characterizes a market scenario that falls within Region (C) of Figure 1, in which $0 < v_{nr} < v_r < v_p < 1$ is the ensuing equilibrium outcome. The results in Proposition 4 can be observed in the range $0 < \delta < 0.204$ in Figure 4. The vendor benefits from an increase in residual loss factor δ for the same reason for which he benefits when R increases (as discussed in part (b) of Proposition 1). An increase in δ only directly impacts ransom-paying unpatched consumers, providing a disincentive for them to take that route. As some consumers who were paying ransom switch to patching due to this increased risk of possibly not receiving promised decryption keys, the aggregate risk externality decreases and the vendor can increase the price while keeping the overall market relatively steady to extract additional surplus (as seen in panels (a) and (b) of Figure 4). In short, when all market segments are present in equilibrium, the vendor prefers greater potential residual losses (whether stemming from failures with payment systems or decryption keys as well as mixed motivations of hackers) because it presents an unusual and counter-intuitive opportunity to charge a premium for higher risk in the market without losing too many consumers.

Furthermore, as it turns out, the vendor would prefer that consumers have the worst possible perception regarding the trustworthiness of the attacker whereas an economically-driven hacker would prefer the opposite. Reports suggest that, compared to the early days of ransomware attacks, the market for such attacks has become efficient and the success rate in retrieving access to compromised assets following a ransom payment increased dramatically, highlighting prevalent economic motivations on the attacker side (Disparte 2018). But, in many cases, corporate victims that pay ransom do not publicize their actions (Cimpanu 2017), which makes it easier for the vendor to vilify attackers in an amplified way even when decryption keys are often returned. Even a small number of failed interactions can damage the hacker's reputation and effectively cut off its revenue stream.

Proposition 4 further shows that the vendor's expected profit can be the lowest at the same δ that concomitantly gives the worst expected losses to the unpatched population and the highest overall consumer surplus, as seen in panel (d) of Figure 4. As δ increases, the ransom-paying unpatched population shrinks as customers at both ends of this segment choose different strategies (higher-valuation customers choose to patch, while lower-valuation customers choose not to pay ransom). Moreover, the overall unpatched population shrinks as well, thus lowering the security risk externality. The redistribution of consumers among

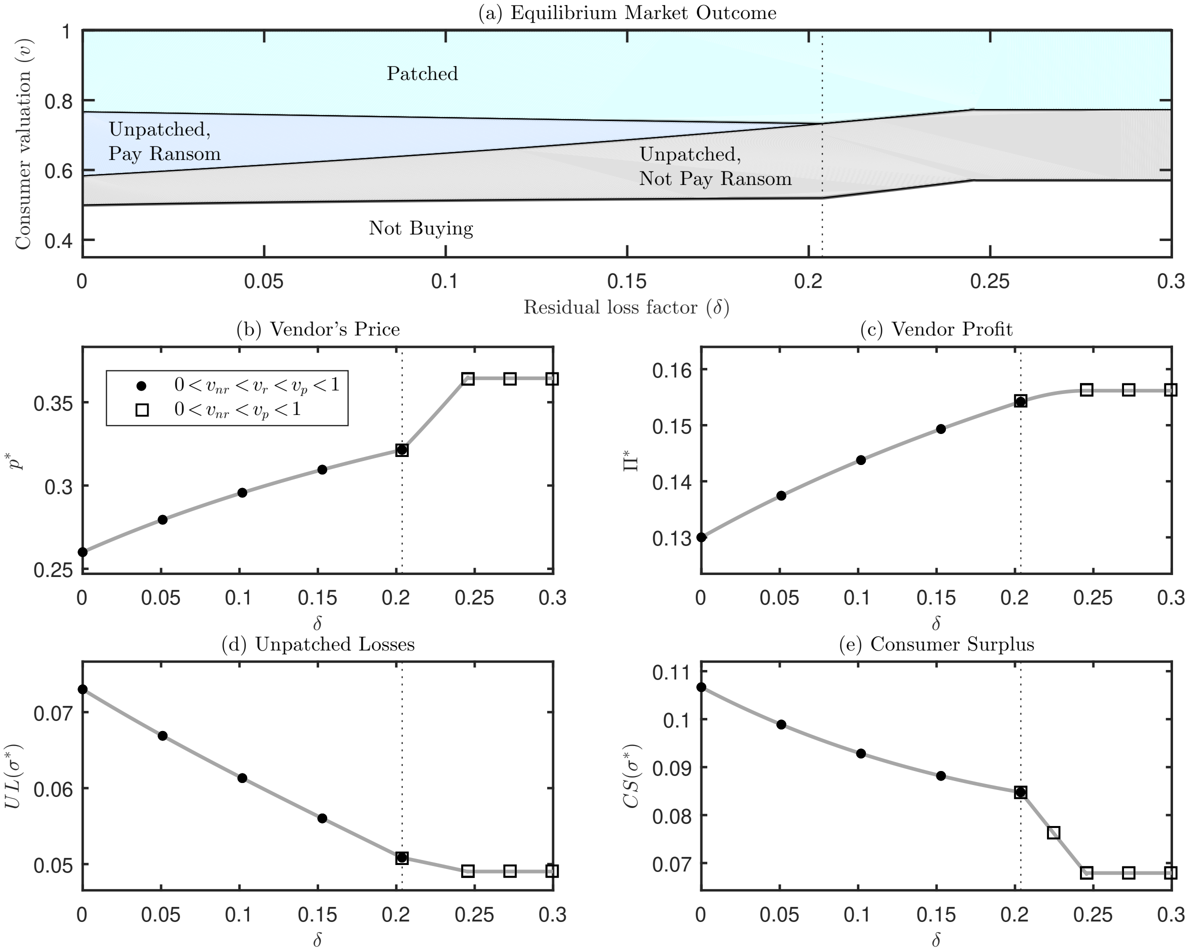

Figure 4: Impact of the residual loss factor (δ) on the equilibrium market outcome, vendor's pricing, vendor's profit, aggregate unpatched consumer losses, and consumer surplus. The parameter values are $c_p = 0.28$, $\alpha = 3$, $R = 1.75$, and $\pi = 0.6$.

segments and the reduced overall risk result in the aggregate expected losses to the unpatched population decreasing in δ. Even though these expected losses to the unpatched population are decreasing, the vendor mitigates the increased risk of residual losses by employing a higher price to control the population size. Given the relatively stable (but slightly shrinking) size of the market when $\delta < 0.204$, the reduction in losses to the unpatched population is offset by the larger premium, hence consumer surplus also decreases in δ.

Having fully explored the case where residual losses for ransom-paying consumers are low, we next turn our attention to the case where residual losses become high. An economically-motivated hacker may aptly be characterized as having a lower δ because revenue generation requires a mass of consumers to pay ransom in equilibrium. In particular, a lower δ helps to make this strategy incentive compatible for some subset of the consumer space. On the other hand, a politically-motivated hacker or a hacker otherwise motivated may be significantly less concerned about capping residual losses. In that generating ransom payments is no longer a primary concern, the practical range of δ for hackers with these motivations may become much broader. In this sense, it is worthwhile to also examine equilibrium outcomes when δ is high and explore how the relevant comparative statics are impacted.

Proposition 5. *If* $\pi\alpha > \frac{2(1-c_p)c_p}{1-3c_p}$ *and* $\delta > 1 - \frac{R}{\alpha}$*, then the equilibrium consumer market structure is* $0 < v_{nr} < v_p < 1$*. The vendor's equilibrium price, the size of the market, and equilibrium profit are all constant in* R *and* δ*.*

Proposition 5 formally demonstrates that as the residual losses become sufficiently high, it is no longer incentive compatible for any user to pay ransom in equilibrium. Rather than pay ransom, users will either remain unpatched and risk losses or incur patching costs to shield them. A higher δ reflects a situation where the attacker tends not to fulfill any agreement to release decryption keys after receiving ransom payments. This undermines the upside of paying ransom and users avoid that strategy in equilibrium. Notably, once the characteristics of the ransomware environment discourage ransom-paying strategies, the equilibrium outcomes necessarily converge to those under the benchmark scenario. As can be seen in the right-hand side of all panels in Figure 4, the relevant measures all become constant in δ once the residual loss factor has become sufficiently high. Because of convergence to the benchmark, one might naturally think of the benchmark instead as the outcome under a politically-motivated hacker. However, in the next section, we will discuss why this might be misleading despite the seeming equivalence between the two in the current cybersecurity landscape.

4.3 Comparison to Benchmark

To help vendors and governments better understand how a software market with ransomware present in the threat landscape is *different* from one where the threat landscape does not give mitigation recourse to victims, we contrast findings from our model against findings from the benchmark model introduced in Equation (2). In order for the comparison to be meaningful, we focus on sensitivity analysis with respect to a common primitive shared by the two models, namely the security risk factor, π. By comparing outcomes between the two scenarios given a common risk factor, we focus in both cases on similar attack vectors for infiltration and spread within networks. As such, any difference in the nature and magnitude of outcomes is attributable to the additional option for consumers to pay ransom and the strategic behavior that results from its presence.

4.3.1 Low Residual Loss Factor (δ)

We begin with the case of low δ which represents ransomware designed with revenue generation in mind. First, we explore how the vendor's optimal pricing strategy is different under the two scenarios.

Proposition 6. *There exists bounds $\tilde{\delta} > 0$ and $0 < \hat{\pi} < \frac{c_p \alpha}{R^2 - c_p R \alpha}$ such that if $\delta < \tilde{\delta}$ and $\frac{\alpha}{2 - c_p} < R < \tilde{R}$:*

(a) if $\max\left(\frac{\alpha - 2R}{3R^2 - 4R\alpha + \alpha^2}, \frac{2c_p}{\alpha}\right) < \pi < \hat{\pi}$, then $p^*_{RW} > p^*_{BM}$;

(b) if $\hat{\pi} < \pi < 1$, then $p^*_{RW} < p^*_{BM}$;

(c) p^*_{BM} is decreasing in π.

Proposition 6 is illustrated in Figure 5, with the bound $\hat{\pi} \approx 0.36$ for the specified parameter set. Figure 5 depicts a cross-section of Figure 1 in the π-direction at $R = 1.75$. In particular, for π ranging from 0 to 0.6, this slice cuts through Regions (E), (A), and (C) of Figure 1; these region labels are also provided in panel (b) of Figure 5. When the risk factor (π) is sufficiently small, whether ransomware is present in the landscape or not, no customer has any incentive to patch since expected losses are small relative to the cost of patching. In the ransomware scenario, since we are exploring a region with a moderate ransom demand, the unpatched consumer indifferent between paying ransom and not paying ransom (v_r) derives strictly positive utility in equilibrium. Thus, the emerging market structure is $0 < v_{nr} < v_r < 1$ (as seen in panel (b) of Figure 5). Low-valuation consumers who adopt under the given risk circumstances face expected losses that are small enough such that paying ransom is not incentive-compatible for them. Hence, the lowest-valuation adopter under ransomware scenario does not factor in the ransom demand in her utility, although she is affected by the overall unpatched population size. Since the vendor cares about the entire adopter population, the optimal price, profit, and total market size under both ransomware and benchmark scenarios are the same in this region. This can be seen in Figures 5 and 6.

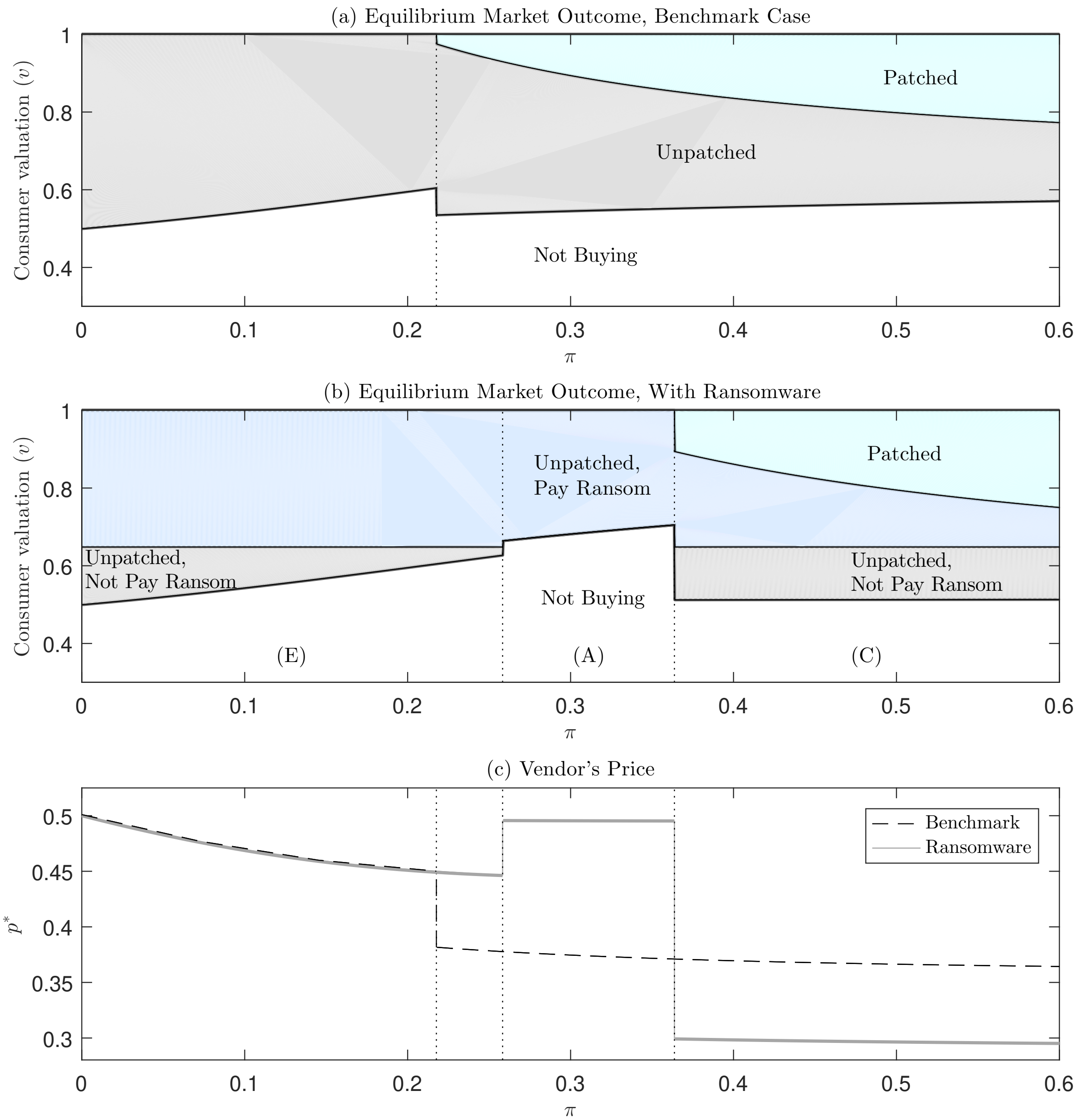

Figure 5: Sensitivity of equilibrium consumer market structure and price with respect to risk factor π under both benchmark and ransomware cases. The parameter values are $c_p = 0.28$, $\alpha = 3$, $\delta = 0.1$, and $R = 1.75$.

In particular, panels (a) and (b) of Figure 6 show the impact of π on the vendor's profit and equilibrium market size, $M(\sigma^*) \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v) \in \{(B,NP,NR),(B,NP,R),(B,P)\}\}}\, dv$.

As π increases into moderate and high ranges, we see differences in pricing strategy (and corresponding market outcomes) between the two threat landscapes. While in both scenarios the price remains piece-wise decreasing, the two pricing strategies present jumps

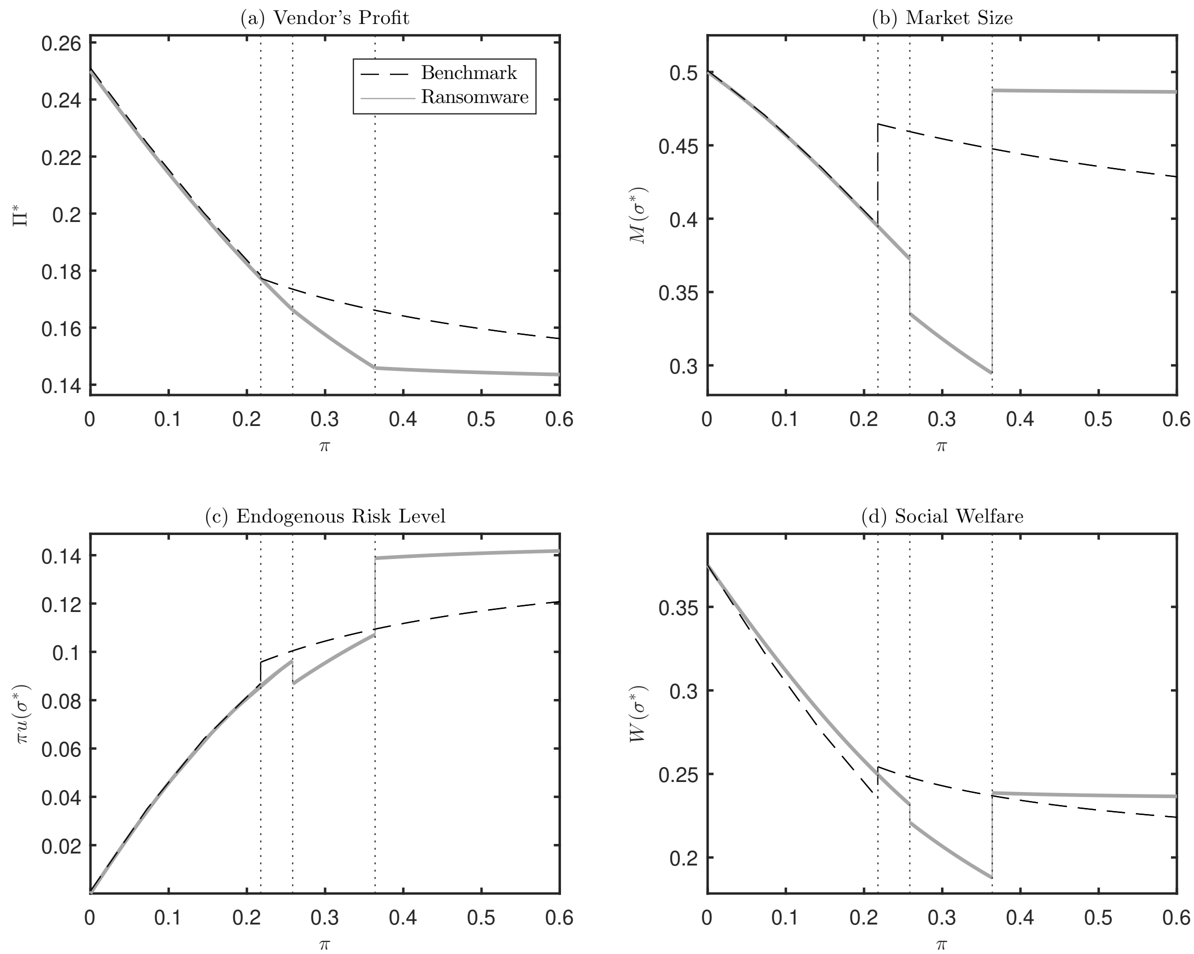

Figure 6: Sensitivity of vendor's profit, consumer market size, endogenous total risk level (πu), and social welfare in equilibrium with respect to the risk factor (π) under both benchmark and ransomware scenarios. The parameter values are $c_p = 0.28$, $\alpha = 3$, $\delta = 0.1$ and $R = 1.75$.

at points of discontinuity (corresponding to changes in market structure) that highlight significant differences in the vendor's approach toward mitigating risk using price. In the benchmark case, when the risk factor level first reaches a moderate level (around $\pi \approx 0.22$), the endogenous overall total security risk πu in equilibrium is also relatively high (as can be seen from panel (c) of Figure 6). At such a risk level, the vendor has an incentive to significantly drop his price (a discontinuous reduction, illustrated in panel (c) of Figure 5) in a strategic manner to profitably increase the market size. Facing a larger unpatched population, the highest-valuation consumers opt to patch, which insulates them from the added externality introduced by more lower-valuation consumers joining the market but not patching. However, in the vicinity of $\pi = 0.22$, this dynamic does not occur in the ransomware case. In particular, if the vendor were to drop the price, some of the highest-valuation consumers would not patch, even when facing this increased risk. Instead, they would still opt to just pay the ransom and bear the valuation-dependent losses $\delta\alpha v$. As long as these highest-valuation customers remain unpatched, they continue to impose a negative externality on all other unpatched customers in the market. Because of that, the vendor cannot profitably expand the market through a significant drop in price, and the optimal price in the ransomware case stays above the price of the benchmark case.

In a more striking contrast to the benchmark case, the vendor in the ransomware case actually has an incentive to hike the price altogether to a higher range (still keeping price piecewise decreasing in π) as risk increases further. This happens for π between 0.26 and 0.36, as can be seen in panel (c) of Figure 5. This outcome and the dynamics at play which drive it have been analyzed in Proposition 2 and the related discussion. Notably, in the benchmark case, the vendor never hikes the price when changing the market structure from $0 < v_{nr} < 1$ to $0 < v_{nr} < v_p < 1$ as the security risk increases. As risk increases, lower-valuation consumers who do not patch are strongly impacted because there is not an option to pay ransom in order to mitigate losses. As such, a price increase would hurt the unpatched population even more, leading to a significant drop in market size and a lower profit.

Once the risk factor becomes sufficiently high, then the vendor drops price significantly in the ransomware case, even below the benchmark level. This happens around $\pi = 0.36$ in panel (c) of Figure 5. This move expands the market significantly at the low end, inviting a large mass of unpatched customers to join the market (with a sizable fraction of the lowest-valuation consumers opting to not pay the ransom if hit), as seen in panel (b) of Figure 5. This behavior introduces significant overall risk to the market which is depicted in panel (c) of Figure 6. As a result of this increase in the endogenous aggregate risk, high-valuation consumers find it optimal to protect themselves from this risk by patching. Consequently, the resulting consumer market equilibrium outcome is $0 < v_{nr} < v_r < v_p < 1$, which most closely resembles today's software markets.

Interestingly, panel (a) of Figure 6 shows that, once the risk factor is high enough ($\pi > 0.22$), the vendor will be strictly better off under the benchmark model than under the ransomware model. When π ranges between 0.22 and 0.26, under ransomware, the market keeps shrinking even though the price keeps dropping. At the same time, under the benchmark, the big drop in price induces a significant increase in market size. This enables the vendor to obtain higher profits. When π ranges from 0.26 to 0.36, under ransomware, the vendor significantly increases price so that risk from the unpatched population is mitigated. In this region, nobody patches and the risk is too high - the vendor is better off shrinking the consumer population rather than lowering the price. Moreover, the unpatched group that pays ransom is highly elastic to risk in this region. On the other hand, under the benchmark, the overall population remains relatively stable (only slightly decreasing), as the presence of security risk alters the sizes of the patched and unpatched groups in opposite directions in a balanced way. With prices relatively inelastic in this region, the profit under the benchmark is superior.

When the risk factor is even higher ($\pi > 0.36$), the gap between the two profit levels is shrinking but still the profit under the benchmark equilibrium dominates. Under ransomware, in that there are two unpatched groups (those who pay ransom and those who do not), efforts to increase adoption via lower prices will be associated with very high aggregate risk (πu). Thus, the reduction in price must be significantly greater for users to bear these risks making it also much lower than the price employed in the benchmark case. This prevents it from reaching the same profitability as the benchmark case. We also note that the vendor's profit is more elastic with respect to risk under the benchmark when $\pi \in (0.22, 0.36)$ compared to the ransomware case, but the ordering changes for higher risk levels ($\pi > 0.36$). This suggests that the vendor has less incentive to improve security under the benchmark compared to ransomware when risk is moderate, and has greater incentive to do so when risk is high. Taken together with prior observations, consumers in today's ransomware threat landscape should be more mindful of the risks of remaining unpatched not only because of the increased externality arising from a sizable unpatched population but also because of decreased incentives by the vendor to produce secure software.

Lastly, we study the impact of ransomware on social welfare. In the ransomware case, we denote the expected aggregate security attack losses incurred by the unpatched population not paying ransom by $NL_{RW} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,NR)\}} \pi \alpha u(\sigma^*) v dv$. Similarly, we denote the expected aggregate losses incurred by the ransom-paying unpatched population by $RL_{RW} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,R)\}} \pi u(\sigma^*)(R + \delta\alpha v) dv$, and the aggregate expected patching costs by $PL_{RW} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,P)\}} c_p dv$. Accounting for all of these components, the total expected security-related losses can be expressed as: $L_{RW} \triangleq NL_{RW} + RL_{RW} + PL_{RW}$. Social welfare is then given by $W_{RW} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)\in\{(B,NP,NR),(B,NP,R),(B,P)\}\}} v dv - L_{RW}$. Similarly, for the benchmark case, we define $NL_{BM} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP)\}} \pi\alpha u(\sigma^*) v dv$, $PL_{BM} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,P)\}} c_p dv$, $L_{BM} \triangleq NL_{BM} + PL_{BM}$, and $W_{BM} \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)\in\{(B,NP),(B,P)\}\}} v dv - L_{BM}$. The presence of an option to pay ransom can have a significant effect on the market structure, impacting welfare in complex ways, which we illustrate in the next result.

Proposition 7. *There exist bounds $\tilde{\delta}, \tilde{\pi}_L, \underset{\sim}{\pi}_M, \tilde{\pi}_M, \underset{\sim}{\pi}_H > 0$ satisfying the condition $0 < \tilde{\pi}_L < \underset{\sim}{\pi}_M < \tilde{\pi}_M < \frac{c_p\alpha}{R^2 - c_p R\alpha} < \underset{\sim}{\pi}_H < 1$, such that if $\delta \le \tilde{\delta}$ and $\frac{\alpha}{2-c_p} < R < \tilde{R}$:*

(a) if $0 < \pi < \tilde{\pi}_L$, then $W_{RW} > W_{BM}$;

(b) if $\underset{\sim}{\pi}_M < \pi < \tilde{\pi}_M$, then $W_{RW} < W_{BM}$;

(c) if $\tilde{\pi}_H < \pi < 1$, then $W_{RW} \ge W_{BM}$.

When risk is sufficiently low, consumers do not have strong incentives to patch. Whether they have the option to pay ransom or not, consumers remain unpatched and, as discussed before, the optimal price and market size are identical under the two scenarios. This can be see in panels (b) and (c) of Figure 6 for $\pi < 0.22$. The only difference is that, in the ransomware case, unpatched consumers with higher valuation manage to counter the potentially high valuation-dependent losses by paying ransom. Therefore, the overall losses are lower in the ransomware case, which leads to higher social welfare as can be seen in panel (d) of Figure 6.

Social welfare is also higher in the ransomware case when risk is high ($\pi > 0.36$) but for a different reason. In this region, high-valuation consumers patch under both scenarios. However, under ransomware, the vendor employs a significantly lower price and achieves a larger market size, including a larger unpatched population. Nevertheless, some of the unpatched consumers are able to reduce their losses by paying ransom instead. These two effects combined lead to higher social welfare under ransomware in comparison to the benchmark. Notably, in the high risk region, the vendor's interests are not aligned with the scenario yielding higher social welfare. The vendor actually prefers that consumers do not have the recourse of paying ransom. This in turn places additional pressure on consumers, leading to a reduced unpatched population, and ultimately enabling the vendor to charge a higher price.

When the risk is within an intermediate range (π between 0.22 and 0.36 in Figure 5), the vendor sets a higher price in the ransomware case compared to the benchmark, resulting in a market size that is significantly lower under ransomware. While most (to all) unpatched consumers pay ransom and nobody patches, the significant difference in market size between the ransomware and benchmark cases ensures that social welfare is higher in the latter case, matching also the vendor's scenario preference.

4.3.2 High Residual Loss Factor (δ)

We turn our attention to residual losses being high (i.e., characterized by a high δ) which has been representative of politically-motivated ransomware attacks such as WannaCry (Greenberg 2018). As we saw in Section 4.2, once δ becomes high, users no longer have sufficient incentive to pay ransom. Therefore, the remaining feasible strategies for users match those under the benchmark scenario, which gives rise to equivalent equilibrium measures in both cases. In particular, if $\delta > 1 - \frac{R}{\alpha}$, then $p^*_{RW} = p^*_{BM}$. And consequently, $\Pi^*_{RW} = \Pi^*_{BM}$ and $W_{RW} = W_{BM}$.

Viewed differently, if politically-motivated hackers are utilizing ransomware attacks with high residual losses, then they need not employ ransomware at all; traditional attacks result in equivalent outcomes. However, given that high δ ransomware is actually deployed, this equivalence brings forth the question of whether such ransomware could be even more harmful depending on the motivation of the hacker. For example, if a politically-motivated hacker targeted total losses associated with the software, then lowering δ to induce some ransom payments would actually be more effective. Panel (a) of Figure 7 illustrates a case where lowering δ to a medium range, i.e., $\delta \in (0.42, 0.56)$, increases expected losses relative to the higher range, i.e., $\delta > 0.56$.

In contrast, at a higher level of ransom demand, a politically-motivated hacker focused on total expected losses might benefit from a minimal δ which greatly boosts total expected losses as is illustrated in panel (c) of Figure 7. In this case, the behavior of politically-motivated and economically-motivated hackers actually coincides unlike in the case of a lower ransom demand shown in panel (a). On the other hand, as we discussed earlier, it is not easy to translate *politically-motivated* to an objective goal. For instance, perhaps politically-motivated could instead mean focused on reducing social welfare. In that case, a high level of residual losses where equivalence with the benchmark is achieved would be more

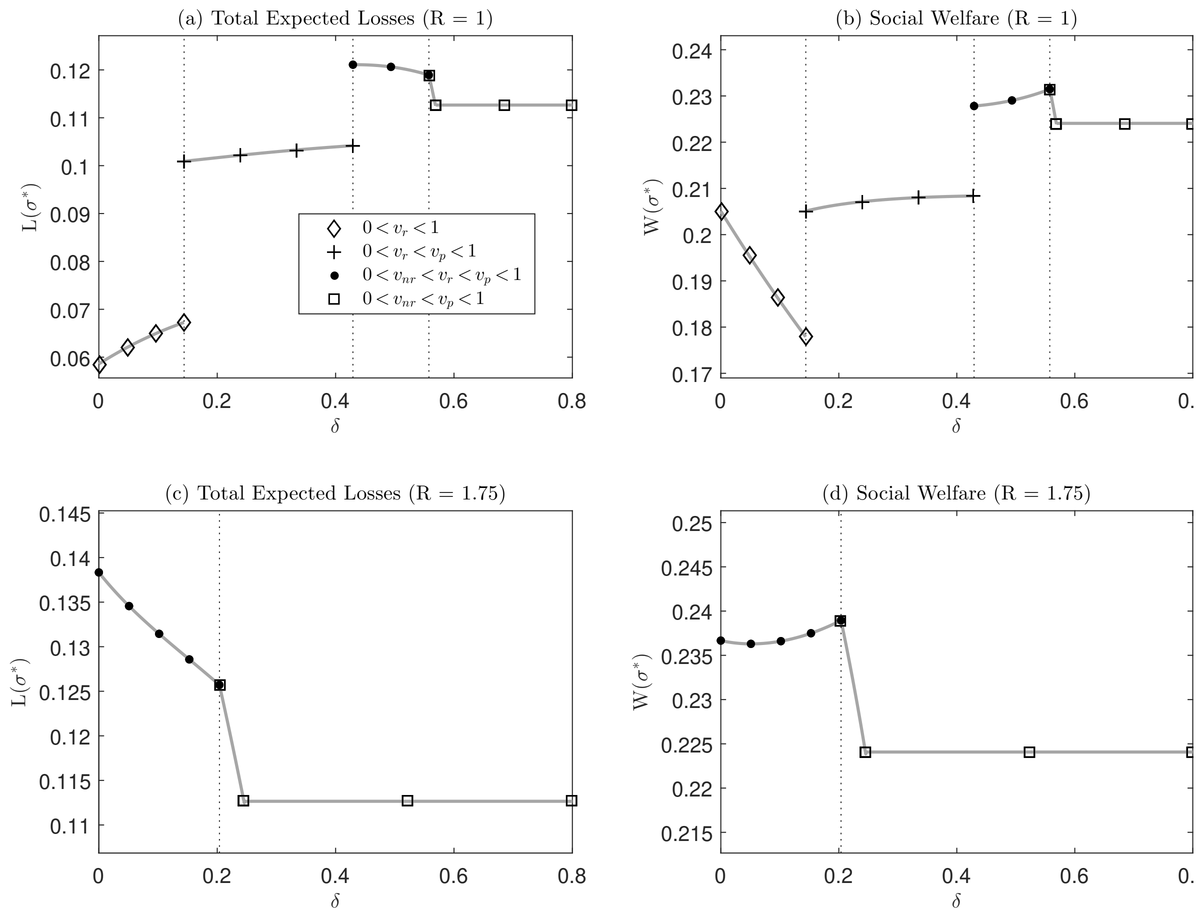

Figure 7: Impact of residual loss factor (δ) on total expected losses and social welfare. The common parameter values are $c_p = 0.28$, $\alpha = 3$, and $\pi = 0.6$. Panels (a) and (b) illustrate comparative statics for $R = 1$, while panels (c) and (d) do the same for $R = 1.75$.

effective at reducing welfare, which is illustrated in panel (d) of Figure 7. This case highlights the inherent difficulty with hacker motivations: one gets polarizing predictions depending on how even a single motivation (such as being political) gets operationalized. Layering on that there are many diverse hacker motivations, this issue gets further compounded. In light of these issues, our paper aimed to cover and provide insights across a broader set of these motivations by exploring varying levels of R and δ in different regimes. Panel (b) of Figure 7 underscores the complexity that arises by depicting how a lower boundary value of δ is the most effective at reducing welfare for a lower level of ransom demand.

5 Conclusion

With the rise of cryptocurrency-based payment systems, malicious hackers are finding it increasingly profitable to conduct ransomware attacks. While traditional ransomware attacks have been targeted in nature, modern ransomware variants are less targeted and instead exhibit the capability to spread laterally across unprotected systems leading to large scale reach and damage. In this paper, we study the impact of ransomware attacks on software markets. The presence of ransomware in a software product's threat landscape can qualitatively change the nature of the consumer market structure. In particular, by giving victims an opportunity to mitigate their losses by paying ransom, ransomware operators segment the unpatched consumer population into two interdependent tiers. Ransomware directly impacts the ransom-paying consumer segment while indirectly impacting all market segments through the negative security externality that all unpatched users generate. This segmentation of consumer behavior drives unexpected findings. For example, both the equilibrium market size and the vendor's profit under optimal pricing can actually increase in the ransom demand as well as the risk of residual losses following a ransom payment (which reflects the trustworthiness of the ransomware operator). In such cases, software vendors would prefer for consumers to believe that ransomware attackers are not trustworthy. Moreover, we also show that the expected total ransom paid is non-monotone in the risk of success of the attack, increasing when the risk is moderate in spite of a decreasing ransom-paying population.

In order to properly assess the market changes induced by the option to pay ransom, we also compare and contrast market outcomes in the ransomware case to similar outcomes under a benchmark scenario where consumers do not have the option to mitigate the losses by paying ransom. For intermediate levels of risk, the vendor under the ransomware case restricts software adoption by hiking the price to a significantly high level. This lies in stark contrast to outcomes in the benchmark case where the vendor always decreases price as security risk increases. While in low and high risk settings, social welfare is higher under the ransomware case compared to the benchmark case, it turns out that for intermediate risks levels, it is better from a social standpoint for consumers not to have an option to pay ransom. When risk is high, a vendor in the ransomware case has incentives to set a significantly lower software price compared to the benchmark case; this pricing behavior leads to greater overall risk for consumers as a mass of low-valuation customers enter the market and choose to remain unpatched. While this market expansion is better for social welfare overall, consumers in today's world end up bearing more risk in a market with ransomware due to increased unpatched usage.

When the hackers are unreliable (or politically-motivated) or the ransom demand is too high, the ransomware and benchmark cases match in terms of outcomes. There are a myriad of agendas that could motivate a hacker. Nevertheless, as we reveal in our discussion, when the ransom demand level is high enough, a hacker might be able to exact the maximum level of expected loss on the consumer population in the scenario of no residual losses (if paying ransom). Also, when the ransom demand is low, a hacker could exact the maximum social welfare loss at a relatively low level of residual loss. Taken together, these findings paint an unexpected picture - hackers determined to exact maximum damage on society may actually be inclined to help all victims who pay ransom to recover access to their compromised assets. To unsuspecting victims, their propensity to facilitate a successful restoration of assets may make them seem solely economically driven. These dynamics are driven by the network security externalities characteristic to ransomware worms. Revisiting the argument from the introduction, in the case of NotPetya and WannaCry, which correspond to high δ scenarios, our analysis shows that it is possible that the outcome of the attacks could have been even more destructive if actually some of the victims could have recovered access to their digital assets by paying the ransom.

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Appendix

A Consumer Market Equilibrium

Lemma A.1. *The complete threshold characterization of the consumer market equilibrium is as follows:*

(I) $(0 < v_{nr} < 1)$, *where* $v_{nr} = \frac{\pi\alpha - 1 + \sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha}$*:*

- *(A)* $p < 1$
- *(B)* $R \geq \alpha(1-\delta)$
- *(C)* $1 + \pi\alpha \leq 2c_p + \sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}$

(II) $(0 < v_{nr} < v_p < 1)$, *where* v_{nr} *is the most positive root of the cubic* $f_1(x) \triangleq \pi\alpha x^3 + (1-(c_p+p)\pi\alpha)x^2 - 2px + p^2$ *and* $v_p = v_{nr} + \frac{v_{nr}-p}{\pi\alpha v_{nr}}$*:*

- *(A)* $c_p\alpha(R - c_p\alpha(1-\delta))(1-\delta)^2 \leq R^2(R - \alpha(c_p+p)(1-\delta))\pi$
- *(B)* $R - c_p\alpha(1-\delta) > 0$
- *(C)* $(-1+c_p+p)\pi\alpha < -c_p + c_p^2$

(III) $(0 < v_{nr} < v_r < 1)$, *where* $v_{nr} = \frac{\pi\alpha - 1 + \sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha}$ *and* $v_r = \frac{R}{\pi\alpha(1-\delta)}$*:*

- *(A)* $p > 0$
- *(B)* $-2R\pi + (1-\delta)(-1+\pi\alpha+\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}) < 0$
- *(C)* $R < \alpha(1-\delta)$
- *(D)* $2c_p\alpha + (R+\alpha\delta)(\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)} - (1+\pi\alpha)) \geq 0$

(IV) $(0 < v_{nr} < v_r < v_p < 1)$, *where* v_{nr} *is the most positive root of the cubic* $f_2(x) \triangleq \delta\pi\alpha x^3 + (\delta + R\pi - (c_p+p\delta)\pi\alpha)x^2 - (2\delta+R\pi)px + p^2\delta$, $v_r = \frac{R}{\pi\alpha(1-\delta)}$, *and* $v_p = v_{nr} + \frac{v_{nr}-p}{\pi\alpha v_{nr}}$*:*

- *(A)* $R > p\alpha(1-\delta)$
- *(B)* $R^2(R-(c_p+p)\alpha(1-\delta))\pi > -(R-p\alpha(1-\delta))^2\delta(1-\delta)$
- *(C)* $\alpha(c_p-\delta) + \alpha^2\pi(c_p(\pi\alpha+2p-2) + \delta(\alpha(p-1)\pi - 3p+2)) + \sqrt{\pi\alpha(\pi\alpha+4p-2)+1}(\alpha(\pi\alpha(c_p+\delta(p-1)) - c_p+\delta) + \alpha(p-1)\pi R + R) + R(\pi\alpha(\alpha(p-1)\pi - 3p+2) - 1) < 0$
- *(D)* *Either* $\Bigg((\alpha c_p(\delta-1)^3(2\pi\alpha p+1) + \sqrt{(\delta-1)^2 + 2(\delta-1)\pi(2\alpha(\delta-1)p+R) + \pi^2R^2} \times$

  $(\alpha c_p(\delta-1)^2 + (\delta-1)R(\pi\alpha(c_p+p)+1) + \pi R^2) + (\delta-1)\pi R^2(\pi\alpha(c_p+p)+2) +$

$(\delta-1)^2R(2\pi\alpha c_p+3\pi\alpha p+1)+\pi^2R^3<0\big)$ *and*
$\big(\delta+\sqrt{4\alpha(\delta-1)^2p\pi+(\delta+\pi R-1)^2}<\pi R+1\big)\Big)$, *or*

$\Big(\big(\alpha c_p(\delta-1)^3(2\pi\alpha p+1)+\sqrt{(\delta-1)^2+2(\delta-1)\pi(2\alpha(\delta-1)p+R)+\pi^2R^2}\times$
$(-\alpha c_p(\delta-1)^2-(\delta-1)R(\pi\alpha(c_p+p)+1)-\pi R^2)+(\delta-1)\pi R^2(\pi\alpha(c_p+p)+2)+$
$(\delta-1)^2R(2\pi\alpha c_p+3\pi\alpha p+1)+\pi^2R^3>0\big)$ *and*
$\big(\pi(R-2\alpha(1-\delta)p)>-\delta+\sqrt{4\alpha(\delta-1)^2p\pi+(\delta+\pi R-1)^2}+1\big)\Big)$

(V) $(0<v_r<1)$, *where* $v_r=\frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}$:

(A) $p<1$

(B) $c_p\left(\alpha\delta\pi+\sqrt{2\pi(\alpha\delta(2p-1)+R)+\pi^2(\alpha\delta+R)^2+1}+\pi R+1\right)\geq 2(1-p)\pi(\alpha\delta+R)$

(C) *Either* $\left(2\delta+\sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2}\leq\pi\alpha\delta+\pi R+1\right)$, *or*

$\Big(2\delta+\sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2}>\pi\alpha\delta+\pi R+1$ *and*
$\frac{\pi\alpha\delta+\sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2}-R\pi-1}{2\pi\alpha\delta}\leq$
$-\frac{2\delta p}{\pi\alpha\delta-2\delta-\sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2}+\pi R+1}\Big)$

(VI) $(0<v_r<v_p<1)$, *where* v_r *is the most positive root of the cubic* $f_3(x)=\alpha^2\delta^2\pi x^3+(\alpha\delta(1+2R\pi-(c_p+p)\delta\pi\alpha))x^2+(R^2\pi-2\alpha\delta(p+(c_p+p)R\pi))x+p^2\alpha\delta-(c_p+p)R^2\pi$ *and* $v_p=v_r+\frac{v_r-p}{(R+v_r\alpha\delta)\pi}$:

(A) $\left(\alpha\delta\pi+\sqrt{4\alpha\delta\pi(p+\pi R)+(-\alpha\delta\pi+\pi R+1)^2}+\pi R-1\right)^2\times$
$\left(\pi(-(\alpha\delta(2c_p+2p-1)+R))+\sqrt{4\alpha\delta\pi(p+\pi R)+(-\alpha\delta\pi+\pi R+1)^2}-1\right)+$
$2\left(\alpha\delta\pi-2\alpha\delta p\pi+\sqrt{4\alpha\delta\pi(p+\pi R)+(-\alpha\delta\pi+\pi R+1)^2}-R\pi-1\right)^2>0$

(B) $\frac{\alpha\delta\pi+\sqrt{4\alpha\delta\pi(p+\pi R)+(-\alpha\delta\pi+\pi R+1)^2}-R\pi-1}{2\alpha\delta\pi}>p$

(C) *Either* $\left(\pi\alpha c_p+\delta^2\geq\alpha\delta\pi(c_p+p)+\delta+\pi R\right)$, *or*

$\Big(\big(\pi\alpha c_p+\delta^2<\alpha\delta\pi(c_p+p)+\delta+\pi R\big)$ *and*
$\Big(\big(\frac{R}{\alpha(1-\delta)}\leq p\big)$ *or*
$\big(\frac{R}{\alpha(1-\delta)}>p$ *and* $\pi R^2(\alpha(\delta-1)(c_p+p)+R)\leq(\delta-1)\delta(\alpha(\delta-1)p+R)^2$ *and*
$\frac{R}{\alpha-\alpha\delta}<c_p+p\big)\Big)\Big)$

Proof of Lemma A.1: First, we establish the general threshold-type equilibrium structure. Given the size of unpatched user population u, the net payoff of the consumer with type v for strategy profile σ is written as

$$U_{RW}(v,\sigma) \triangleq \begin{cases} v-p-c_p & if \quad \sigma(v)=(B,P)\,; \\ v-p-\pi u(\sigma)(R+\delta\alpha v) & if \quad \sigma(v)=(B,NP,R)\,; \\ v-p-\pi\alpha u(\sigma)v & if \quad \sigma(v)=(B,NP,NR)\,; \\ 0 & if \quad \sigma(v)=(NB,NP)\,, \end{cases} \tag{A.1}$$

where

$$u_{RW}(\sigma) \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma(v)\in\{(B,NP,R),(B,NP,NR)\}\}}\,dv\,. \tag{A.2}$$

Note $\sigma(v)=(B,P)$ if and only if

$$v-p-c_p \geq v-p-\pi u(\sigma)(R+\delta\alpha v) \Leftrightarrow v \geq \frac{c_p - R\pi u(\sigma)}{\delta\pi\alpha u(\sigma)}, \text{ and}$$

$$v-p-c_p \geq v-p-\pi\alpha u(\sigma)v \Leftrightarrow v \geq \frac{c_p}{\pi\alpha u(\sigma)}, \text{ and}$$

$$v-p-c_p \geq 0 \Leftrightarrow v \geq c_p+p,$$

which can be summarized as

$$v \geq \max\left(\frac{c_p - R\pi u(\sigma)}{\delta\pi\alpha u(\sigma)}, \frac{c_p}{\pi\alpha u(\sigma)}, c_p+p\right). \tag{A.3}$$

By (A.3), if a consumer with valuation v_0 buys and patches the software, then every consumer with valuation $v > v_0$ will also do so. Hence, there exists a threshold $v_p \in (0,1]$ such that for all $v \in \mathcal{V}$, $\sigma^*(v)=(B,P)$ if and only if $v \geq v_p$. Similarly, $\sigma(v) \in \{(B,P),(B,NR),(B,R)\}$, i.e., the consumer of valuation v purchases one of these alternatives, if and only if

$$v-p-c_p \geq 0 \Leftrightarrow v \geq c_p+p, \text{ or}$$

$$v-p-\pi\alpha u(\sigma)v \geq 0 \Leftrightarrow v \geq \frac{p}{1-\pi\alpha u(\sigma)}, \text{ or}$$

$$v-p-\pi u(\sigma)(R+\delta\alpha v) \geq 0 \Leftrightarrow v \geq \frac{p+R\pi u(\sigma)}{1-\delta\pi\alpha u(\sigma)},$$

which can be summarized as

$$v \geq \min\left(c_p+p, \frac{p}{1-\pi\alpha u(\sigma)}, \frac{p+R\pi u(\sigma)}{1-\delta\pi\alpha u(\sigma)}\right). \tag{A.4}$$

Let $0 < v_1 \leq 1$ and $\sigma^*(v_1) \in \{(B,P),(B,NR),(B,R)\}$, then by (A.4), for all $v > v_1$, $\sigma^*(v) \in \{(B,P),(B,NR),(B,R)\}$, and hence there exists a $\underline{v} \in (0,1]$ such that a consumer with valuation $v \in \mathcal{V}$ will purchase the software if and only if $v \geq \underline{v}$.

By (A.3) and (A.4), $\underline{v} \leq v_p$ holds. Moreover, if $\underline{v} < v_p$, consumers with types in $[\underline{v}, v_p]$ choose either (B, NR) or (B, R). A purchasing consumer with valuation v will prefer (B, R) over (B, NR) if and only if

$$v - p - \pi u(\sigma)(R + \delta\alpha v) \geq v - p - \pi\alpha u(\sigma)v \Leftrightarrow v \geq \frac{R}{\alpha(1-\delta)}. \tag{A.5}$$

Next, we characterize in more detail each outcome that can arise in a consumer market equilibrium, as well as the corresponding parameter regions. For Case (A.1), in which all consumers who purchase choose to be unpatched and not pay ransom, i.e., $0 < v_{nr} < 1$, based on the threshold-type equilibrium structure, we have $u(\sigma) = 1 - v_{nr}$. We prove the following claim related to the corresponding parameter region in which Case (A.1) arises.

Claim 1. *The equilibrium that corresponds to case (A.1) arises if and only if the following conditions are satisfied:*

$$p < 1 \text{ and } R \geq \alpha(1-\delta) \text{ and } 1 + \pi\alpha \leq 2c_p + \sqrt{1 + \pi\alpha(-2 + 4p + \pi\alpha)}. \tag{A.6}$$

The consumer indifferent between not purchasing at all and purchasing and remaining unpatched, v_{nr}, satisfies $v_{nr} - p - \pi\alpha u(\sigma)v_{nr} = 0$. To solve for the threshold v_{nr}, using $u(\sigma) = 1 - v_{nr}$, we solve

$$v_{nr} = \frac{p}{1 - \pi\alpha u(\sigma)} = \frac{p}{1 - \pi\alpha(1 - v_{nr})}. \tag{A.7}$$

For this to be an equilibrium, we have that $v_{nr} \geq 0$. This rules out the smaller root of the quadratic as a solution. Given the underlying model assumptions, the other root is strictly positive, so the root characterizing v_{nr} is

$$v_{nr} = \frac{\pi\alpha - 1 + \sqrt{1 + \pi\alpha(-2 + 4p + \pi\alpha)}}{2\pi\alpha} \tag{A.8}$$

For this to be an equilibrium, the necessary and sufficient conditions are that $0 < v_{nr} < 1$, type $v = 1$ weakly prefers (B, NR) to both (B, R) and (B, P).

For $v_{nr} < 1$, it is equivalent to have $p < 1$.

For $v = 1$ to prefer (B, NR) over (B, P), we need $1 \leq \frac{c_p}{\pi\alpha(1-v_{nr})}$. Simplifying, this becomes $1 + \pi\alpha \leq 2c_p + \sqrt{1 + \pi\alpha(-2 + 4p\pi\alpha)}$.

For $v = 1$ to prefer (B, NR) over (B, R), we need $1 \leq \frac{R}{\alpha(1-\delta)}$. Simplifying, this becomes $R \geq \alpha(1-\delta)$. The conditions above are given in (A.6). □

Next, for case (II), in which the lower tier of purchasing consumers is unpatched and does not pay ransom while the upper tier patches, i.e., $0 < v_{nr} < v_p < 1$, we have $u = v_p - v_{nr}$. Following the same steps as before, we prove the following claim related to the corresponding parameter region in which case (II) arises.

Claim 2. *The equilibrium that corresponds to case (II) arises if and only if the following conditions are satisfied:*

$$c_p\alpha(R - c_p\alpha(1-\delta))(1-\delta)^2 \le R^2(R - \alpha(c_p+p)(1-\delta))\pi \text{ and}$$
$$R - c_p\alpha(1-\delta) > 0 \text{ and } (-1+c_p+p)\pi\alpha < -c_p + c_p^2. \quad (A.9)$$

To solve for the thresholds v_{nr} and v_p, using $u = v_p - v_{nr}$, note that they solve

$$v_{nr} = \frac{p}{1-\pi\alpha(v_p - v_{nr})}, \text{ and} \quad (A.10)$$

$$v_p = \frac{c_p}{\pi\alpha(v_p - v_{nr})}. \quad (A.11)$$

Solving for v_p in terms of v_{nr} in (A.10), we have

$$v_p = v_{nr} + \frac{v_{nr} - p}{\pi\alpha v_{nr}}. \quad (A.12)$$

Substituting this into (A.11), we have that v_{nr} must be a zero of the cubic equation:

$$f_1(x) \triangleq \pi\alpha x^3 + (1 - \pi\alpha(c_p+p))x^2 - 2px + p^2. \quad (A.13)$$

To find which root of the cubic v_{nr} must be, note that the cubic's highest order term is $\pi\alpha x^3$, so $\lim_{x\to-\infty} f_1(x) = -\infty$ and $\lim_{x\to\infty} f_1(x) = \infty$. We find $f_1(0) = p^2 > 0$, and $f_1(p) = -c_p\pi\alpha p^2 < 0$. Since $v_{nr} - p > 0$ in equilibrium, we have that v_{nr} is uniquely defined as the largest root of the cubic, lying past p. Then (A.12) characterizes v_p.

For this to be an equilibrium, the necessary and sufficient conditions are $0 < v_{nr} < v_p < 1$ and type $v = v_p$ weakly prefers (B, P) over (B, R). Type $v = v_p$ preferring (B, P) over (B, R) ensures $v > v_p$ also prefer (B, P) over (B, R), by (A.3). Moreover, type $v = v_p$ is indifferent between (B, P) and (B, NR), so this implies that $v = v_p$ weakly prefers (B, NR) over (B, R). This implies that all $v < v_{nr}$ strictly prefer (B, NR) over (B, R) by (A.5).

For $v_p < 1$, first note that from (A.10), we have $\pi\alpha(v_p - v_{nr}) = 1 - \frac{p}{v_{nr}}$ while from (A.11) we have $\pi\alpha(v_p - v_{nr}) = \frac{c_p}{v_p}$. So then solving for v_p, we have

$$v_p = \frac{c_p v_{nr}}{v_{nr} - p}. \quad (A.14)$$

Then using (A.14), a necessary and sufficient condition for $v_p < 1$ to hold is $v_{nr} > \frac{p}{1-c_p}$. This is equivalent to $f_3(\frac{p}{1-c_p}) < 0$, since $\frac{p}{1-c_p} > p$. This simplifies to $(-1+c_p+p)\pi\alpha < -c_p+c_p^2$.

For $v_p > v_{nr}$, no conditions are necessary since v_p was defined in (A.12), and $v_{nr} > p$ by definition of v_{nr} as the largest root of (A.13).

Similarly, $v_{nr} > 0$, by definition of v_{nr}.

To ensure that no consumer has incentive to pay ransom, it suffices to make sure that

$v = v_p$ prefers to not pay ransom over paying ransom. By (A.5), we will need $v_p \leq \frac{R}{\alpha(1-\delta)}$. Using (A.14), this is equivalent to $v_{nr}(R - c_p\alpha(1-\delta)) \geq Rp$. If $R - c_p\alpha(1-\delta) \leq 0$, then no v_{nr} would satisfy this condition in equilibrium. Hence, $R - c_p\alpha(1-\delta) > 0$ is a necessary condition and we need $v_{nr} \geq \frac{Rp}{R-c_p\alpha(1-\delta)}$. This simplifies to $f[\frac{Rp}{R-c_p\alpha(1-\delta)}] \geq 0$, which is equivalent to $c_p\alpha(R - c_p\alpha(1-\delta))(1-\delta)^2 \leq R^2(R - \alpha(c_p+p)(1-\delta))\pi$. The conditions above are summarized in (A.9). □

Next, for case (III), in which there are no patched users while the lower tier chooses to not pay ransom and the upper tier pays ransom, i.e., $0 < v_{nr} < v_r < 1$, we have $u = 1 - v_{nr}$. Following the same steps as before, we prove the following claim related to the corresponding parameter region in which case (III) arises.

Claim 3. *The equilibrium that corresponds to case (III) arises if and only if the following conditions are satisfied:*

$$p > 0 \text{ and } \alpha\big(-2R\pi + (1-\delta)(-1+\pi\alpha+\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)})\big) < 0 \text{ and}$$
$$R < \alpha(1-\delta) \text{ and } 2c_p\alpha + (R+\alpha\delta)(\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)} - (1+\pi\alpha)) \geq 0. \quad \text{(A.15)}$$

To solve for the thresholds v_{nr} and v_r, using $u = 1 - v_{nr}$, note that they solve

$$v_{nr} = \frac{p}{1-\pi\alpha(1-v_{nr})}, \text{ and} \quad \text{(A.16)}$$

$$v_r = \frac{R}{\alpha(1-\delta)}, \quad \text{(A.17)}$$

where the expression in (A.17) comes from (A.5).

Solving for v_{nr} in (A.16), we have

$$v_{nr} = \frac{-1+\pi\alpha \pm \sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha}. \quad \text{(A.18)}$$

Note that $\frac{-1+\pi\alpha-\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha} < p$ while $\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha} > p$, and since $v_{nr} > p$ in equilibrium, it follows that

$$v_{nr} = \frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}}{2\pi\alpha}. \quad \text{(A.19)}$$

For this to be an equilibrium, the necessary and sufficient conditions are $0 < v_{nr} < v_r < 1$ and that type $v = 1$ weakly prefers (B, R) over (B, P). Type $v = 1$ preferring (B, R) over (B, P) ensures $v < 1$ also prefer (B, R) over (B, P), by (A.3). Moreover, type $v = v_r$ is indifferent between (B, R) and (B, NR), so this implies that $v = v_r$ strictly prefers (B, NR) over (B, P). This implies that all $v < v_{nr}$ strictly prefer (B, NR) over (B, P) as well, again by (A.3).

Note $v_{nr} > 0$ is satisfied if $p > 0$ since $v_{nr} > p$ under the preliminary model assumptions.

For $v_{nr} < v_r$, from (A.19) and (A.17), this simplifies to $\alpha\big(-2R\pi + (1-\delta)(-1+\pi\alpha+\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)})\big) < 0$.

For $v_r < 1$, from (A.17), this simplifies to $R < \alpha(1-\delta)$.

For $v = 1$ to weakly prefer (B, R) over (B, P), we need $1 \leq \frac{c_p - R\pi(1-v_{nr})}{\pi\alpha\delta(1-v_{nr})}$. Substituting in (A.19) and simplifying, this becomes $2c_p\alpha+(R+\alpha\delta)(\sqrt{1+\pi\alpha(-2+4p+\pi\alpha)}-(1+\pi\alpha)) \geq 0$. The conditions above are summarized in (A.15). □

Next, for case (IV), in which the top tier of consumers patches, the middle tier pays ransom, and the bottom tier remains unpatched and does not pay ransom, i.e., $0 < v_{nr} < v_r < v_p < 1$, we have $u = v_p - v_{nr}$. Following the same steps as before, we prove the following claim related to the corresponding parameter region in which case (IV) arises.

Claim 4. *The equilibrium that corresponds to case (IV) arises if and only if the following conditions are satisfied:*

$$\begin{gathered}
R > p\alpha(1-\delta) \textit{ and } R^2(R-(c_p+p)\alpha(1-\delta))\pi > -(R-p\alpha(1-\delta))^2\delta(1-\delta) \textit{ and} \\
\alpha(c_p-\delta)+\alpha^2\pi(c_p(\pi\alpha+2p-2)+\delta(\alpha(p-1)\pi-3p+2))+ \\
\sqrt{\pi\alpha(\pi\alpha+4p-2)+1}(\alpha(\pi\alpha(c_p+\delta(p-1))-c_p+\delta)+\alpha(p-1)\pi R+R)+ \\
R(\pi\alpha(\alpha(p-1)\pi-3p+2)-1) < 0 \textit{ and} \\
\textit{either } \Big(\big(\alpha c_p(\delta-1)^3(2\pi\alpha p+1)+\sqrt{(\delta-1)^2+2(\delta-1)\pi(2\alpha(\delta-1)p+R)+\pi^2R^2}\times \\
\big(\alpha c_p(\delta-1)^2+(\delta-1)R(\pi\alpha(c_p+p)+1)+\pi R^2\big)+(\delta-1)\pi R^2(\pi\alpha(c_p+p)+2)+ \\
(\delta-1)^2R(2\pi\alpha c_p+3\pi\alpha p+1)+\pi^2R^3 < 0\big) \textit{ and} \\
\big(\delta+\sqrt{4\alpha(\delta-1)^2p\pi+(\delta+\pi R-1)^2} < \pi R+1\big)\Big), \textit{ or} \\
\Big(\big(\alpha c_p(\delta-1)^3(2\pi\alpha p+1)+\sqrt{(\delta-1)^2+2(\delta-1)\pi(2\alpha(\delta-1)p+R)+\pi^2R^2}\times \\
\big(-\alpha c_p(\delta-1)^2-(\delta-1)R(\pi\alpha(c_p+p)+1)-\pi R^2\big)+(\delta-1)\pi R^2(\pi\alpha(c_p+p)+2)+ \\
(\delta-1)^2R(2\pi\alpha c_p+3\pi\alpha p+1)+\pi^2R^3 > 0\big) \textit{ and} \\
\big(\pi(R-2\alpha(1-\delta)p) > -\delta+\sqrt{4\alpha(\delta-1)^2p\pi+(\delta+\pi R-1)^2}+1\big)\Big).
\end{gathered} \tag{A.20}$$

To solve for the thresholds v_{nr} and v_p, using $u = v_p - v_{nr}$, note that they solve

$$v_{nr} = \frac{p}{1-\pi\alpha(v_p - v_{nr})}, \text{ and} \tag{A.21}$$

$$v_p = \frac{c_p - R\pi(v_p - v_{nr})}{\delta\pi\alpha(v_p - v_{nr})}, \tag{A.22}$$

Note that v_r solves

$$v_r = \frac{R}{\alpha(1-\delta)}, \tag{A.23}$$

where the expression in (A.23) comes from (A.5).

Solving for v_p in (A.21), we have

$$v_p = v_{nr} + \frac{v_{nr} - p}{v_{nr}\pi\alpha}. \tag{A.24}$$

Substituting this into (A.22), we have that v_{nr} must be a zero of the cubic equation:

$$f_2(x) \triangleq \delta\pi\alpha x^3 + (\delta + \pi(R - \alpha(c_p + \delta p)))x^2 - p(2\delta + R\pi)x + p^2\delta. \tag{A.25}$$

To find which root of the cubic v_{nr} must be, note that the cubic's highest order term is $\delta\pi\alpha x^3$, so $\lim_{x\to-\infty} f_2(x) = -\infty$ and $\lim_{x\to\infty} f_2(x) = \infty$. We find $f_2(0) = \delta p^2 > 0$, and $f_2(p) = -c_p\pi\alpha p^2 < 0$. Since $v_{nr} - p > 0$ in equilibrium, we have that v_{nr} is uniquely defined as the largest root of the cubic, lying past p. Then using (A.24), we solve for v_p.

For this to be an equilibrium, the necessary and sufficient conditions are $0 < v_{nr} < v_r < v_p < 1$.

The condition $v_{nr} > 0$ is satisfied without further conditions, since v_{nr} is the largest root of the cubic greater than p by definition.

For $v_{nr} < v_r$ to hold, we need $v_r > p$ and $f_2(v_r) > 0$. These conditions are equivalent to $R > p\alpha(1-\delta)$ and $R^2(R - (c_p + p)\alpha(1-\delta))\pi > -(R - p\alpha(1-\delta))^2\delta(1-\delta)$.

For $v_r < v_p$ to hold, we need $\frac{R}{\alpha(1-\delta)} < v_{nr} + \frac{v_{nr}-p}{\pi\alpha v_{nr}}$ by (A.23) and (A.24). Simplifying, this becomes $(1-\delta)\pi\alpha v_{nr}^2 + (1 - \delta - R\pi)v_{nr} - (1-\delta)p > 0$. Then v_{nr} needs to be larger than the larger root of this quadratic or smaller than the smaller root. The two roots of the quadratic are given by $\frac{-1+\delta+R\pi\pm\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}$. If v_{nr} is larger than the larger root, then a necessary condition is that this larger root is smaller than v_r. On the other hand, if v_{nr} is smaller than the smaller root, then a necessary condition is that the smaller root is larger than p, since by definition $v_{nr} > p$.

Consider the first sub-case in which v_{nr} is larger than the larger root of the quadratic. So then the conditions are $v_{nr} > \frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}$ and $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} < \frac{R}{\alpha(1-\delta)}$. For $v_{nr} > \frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}$, either $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} \le p$, or $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} > p$ and $f_2(\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}) < 0$. The condition $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} \le p$ simplifies to $R \le \alpha p(1-\delta)$. However, $R > p\alpha(1-\delta)$ from $v_r > p$. Since $R > \alpha p(1-\delta)$, a necessary condition is $f_2(\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}) < 0$, which simplifies to $\alpha c_p(\delta-1)^3(2\pi\alpha p+1) + \sqrt{(\delta-1)^2 + 2(\delta-1)\pi(2\alpha(\delta-1)p + R) + \pi^2R^2} \times$

$(\alpha c_p(\delta-1)^2 + (\delta-1)R(\pi\alpha(c_p+p)+1) + \pi R^2) + (\delta-1)\pi R^2(\pi\alpha(c_p+p)+2) + (\delta-1)^2R(2\pi\alpha c_p + 3\pi\alpha p + 1) + \pi^2R^3 < 0$. Lastly for this sub-case, we need that the quadratic root $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} < v_r = \frac{R}{\alpha(1-\delta)}$. This condition simplifies to $\delta + \sqrt{4\alpha(\delta-1)^2p\pi + (\delta + \pi R - 1)^2} < \pi R + 1$. Altogether, these form the first set of conditions in (C) of case (IV).

In the second sub-case in which v_{nr} is smaller than the smaller root of the quadratic, the necessary and sufficient conditions are that $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} > p$ and $v_{nr} < \frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}$. Note that the second condition is equivalent to $f_2(\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}) > 0$ since $f_2(x) > 0$ for any $x > v_{nr}$. The condition that $\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)} > p$ simplifies to $\pi(R - 2\alpha(1-\delta)p) > -\delta + \sqrt{4\alpha(\delta-1)^2 p\pi + (\delta + \pi R - 1)^2} + 1$.

The condition that $f_2(\frac{-1+\delta+R\pi+\sqrt{4p\pi\alpha(1-\delta)^2+(1-\delta-R\pi)^2}}{2\pi\alpha(1-\delta)}) > 0$ simplifies to $\alpha c_p(\delta-1)^3(2\pi\alpha p + 1) + \sqrt{(\delta-1)^2 + 2(\delta-1)\pi(2\alpha(\delta-1)p + R) + \pi^2 R^2} \times (-\alpha c_p(\delta-1)^2 - (\delta-1)R(\pi\alpha(c_p+p)+1) - \pi R^2) + (\delta-1)\pi R^2(\pi\alpha(c_p+p)+2) + (\delta-1)^2 R(2\pi\alpha c_p + 3\pi\alpha p + 1) + \pi^2 R^3 > 0$. Altogether, these form the second set of conditions in (C) of case (IV).

Lastly, we need $v_p < 1$. Using (A.24), this simplifies to $\pi\alpha v_{nr}^2 + (1-\pi\alpha)v_{nr} < p$. Then v_{nr} needs to be between the two roots of that quadratic, $\frac{-1+\pi\alpha\pm\sqrt{1-2\pi\alpha+4p\pi\alpha+(\pi\alpha)^2}}{2\pi\alpha}$. But note that the smaller of the roots not positive, from $0 \leq p \leq 1$ and $\pi\alpha > 0$. Therefore, $v_{nr} > \frac{-1+\pi\alpha-\sqrt{1-2\pi\alpha+4p\pi\alpha+(\pi\alpha)^2}}{2\pi\alpha}$ is satisfied without further conditions. For $v_{nr} < \frac{-1+\pi\alpha+\sqrt{1-2\pi\alpha+4p\pi\alpha+(\pi\alpha)^2}}{2\pi\alpha}$, $f_2(\frac{-1+\pi\alpha+\sqrt{1-2\pi\alpha+4p\pi\alpha+(\pi\alpha)^2}}{2\pi\alpha}) > 0$ is a necessary and sufficient condition since $\frac{-1+\pi\alpha+\sqrt{1-2\pi\alpha+4p\pi\alpha+(\pi\alpha)^2}}{2\pi\alpha} > p$. This simplifies to $\alpha(c_p-\delta)+\alpha^2\pi(c_p(\pi\alpha+2p-2)+\delta(\alpha(p-1)\pi-3p+2)) + \sqrt{\pi\alpha(\pi\alpha+4p-2)+1}(\alpha(\pi\alpha(c_p+\delta(p-1)) - c_p + \delta) + \alpha(p-1)\pi R + R) + R(\pi\alpha(\alpha(p-1)\pi - 3p + 2) - 1) < 0$, which is condition (B) of case (IV). Altogether, the conditions above are given in (A.20). □

Next, for case (V), in which there are no patched users while all consumers who purchase are unpatched and pay ransom, i.e., $0 < v_r < 1$, we have $u = 1 - v_r$. Following the same steps as before, we prove the following claim related to the corresponding parameter region in which case (V) arises.

Claim 5. *The equilibrium that corresponds to case (V) arises if and only if the following* *conditions are satisfied:*

$$\begin{aligned}
&p < 1 \textit{ and}\\
&c_p\left(\alpha\delta\pi + \sqrt{2\pi(\alpha\delta(2p-1)+R) + \pi^2(\alpha\delta+R)^2+1} + \pi R + 1\right) \geq 2(1-p)\pi(\alpha\delta+R),\\
&\textit{and either } 2\delta + \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta+\pi R+1)^2} \leq \pi\alpha\delta + \pi R + 1, \textit{ or}\\
&2\delta + \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta+\pi R+1)^2} > \pi\alpha\delta + \pi R + 1 \textit{ and}\\
&\frac{\pi\alpha\delta + \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta+\pi R+1)^2} - R\pi - 1}{2\pi\alpha\delta} \leq\\
&-\frac{2\delta p}{\pi\alpha\delta - 2\delta - \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta+\pi R+1)^2} + \pi R + 1}. \qquad \text{(A.26)}
\end{aligned}$$

To solve for the thresholds v_r, using $u = 1 - v_r$, note it solves

$$v_r = \frac{p + R\pi(1-v_r)}{1 - \delta\pi\alpha(1-v_r)} \qquad \text{(A.27)}$$

Then v_r is one of the two roots of the equation above, $\frac{-1-R\pi+\delta\pi\alpha\pm\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}$. However, the smaller of the two roots is negative, so v_r must be the larger of the two roots in equilibrium. Hence, we have

$$v_r = \frac{-1 - R\pi + \delta\pi\alpha + \sqrt{4\delta\pi\alpha(p+R\pi) + (1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}. \qquad \text{(A.28)}$$

For this to be an equilibrium, the necessary and sufficient conditions are $p < v_r < 1$, and no consumer prefers to patch or not pay ransom over paying ransom.

For $v_r > p$, using (A.28), this simplifies to $p < 1$. For $v_r > p$, using (A.28), this also simplifies to $p < 1$. Similarly, $v_r < 1$ also simplifies to $p < 1$.

For no consumer to strictly prefer patching over paying ransom, it suffices to have type $v = 1$ weakly prefer paying ransom to patching. This is given as $1 \leq \frac{c_p - R\pi(1-v_r)}{\delta\pi\alpha(1-v_r)}$. Using (A.28), this simplifies to $c_p\left(\alpha\delta\pi + \sqrt{2\pi(\alpha\delta(2p-1)+R) + \pi^2(\alpha\delta+R)^2+1} + \pi R + 1\right) \geq 2(1-p)\pi(\alpha\delta+R)$.

For no consumer to strictly prefer not paying ransom over paying ransom, it suffices to have $v = v_r$ weakly prefer not to buy over buying and not paying ransom (since type $v = v_r$ is indifferent between the option of not purchasing and the option of purchasing, remaining unpatched, and paying ransom). Now if $1 - \pi\alpha u[\sigma] \leq 0$, then $v(1 - \pi\alpha u[\sigma]) - p < 0$, so that everyone would prefer (NB, NP) over (B, NP, NR). In this case, no further conditions are needed. On the other hand, if $1 - \pi\alpha u[\sigma] > 0$, then we will need the condition $v_r \leq \frac{p}{1-\pi\alpha(1-v_r)}$ for $v = v_r$ to weakly prefer not buying over buying but not paying ransom.

In the first sub-case, the condition $v(1 - \pi\alpha u[\sigma]) - p < 0$ simplifies to $2\delta + \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta+\pi R+1)^2} \leq \pi\alpha\delta + \pi R + 1$, using (A.28).

In the second sub-case, the conditions $1 - \pi\alpha u[\sigma] > 0$ and $v_r \leq \frac{p}{1-\pi\alpha(1-v_r)}$ simplify to $2\delta + \sqrt{4\pi\alpha\delta(p+\pi R) + (-\pi\alpha\delta + \pi R + 1)^2} > \pi\alpha\delta + \pi R + 1$ and $\frac{\pi\alpha\delta + \sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2} - R\pi - 1}{2\pi\alpha\delta} \leq -\frac{2\delta p}{\pi\alpha\delta - 2\delta - \sqrt{4\pi\alpha\delta(p+\pi R)+(-\pi\alpha\delta+\pi R+1)^2} + \pi R + 1}$. The conditions above are summarized in (A.26). □

Lastly, for case (VI), in which the top tier patches while lower tier of the market remains unpatched but pays the ransom, i.e., $0 < v_r < v_p < 1$, we have $u = v_p - v_r$. Following the same steps as before, we prove the following claim related to the corresponding parameter region in which case (VI) arises.

Claim 6. *The equilibrium that corresponds to case (VI) arises if and only if the following conditions are satisfied:*

$$\begin{gathered}
\left(\alpha\delta\pi + \sqrt{4\alpha\delta\pi(p+\pi R) + (-\alpha\delta\pi + \pi R + 1)^2} + \pi R - 1\right)^2 \times \\
\left(\pi(-(\alpha\delta(2c_p + 2p - 1) + R)) + \sqrt{4\alpha\delta\pi(p+\pi R) + (-\alpha\delta\pi + \pi R + 1)^2} - 1\right) + \\
2\left(\alpha\delta\pi - 2\alpha\delta p\pi + \sqrt{4\alpha\delta\pi(p+\pi R) + (-\alpha\delta\pi + \pi R + 1)^2} - R\pi - 1\right)^2 > 0 \textit{ and} \\
\frac{\alpha\delta\pi + \sqrt{4\alpha\delta\pi(p+\pi R) + (-\alpha\delta\pi + \pi R + 1)^2} - R\pi - 1}{2\alpha\delta\pi} > p \textit{ and either} \\
\left(\pi\alpha c_p + \delta^2 \geq \alpha\delta\pi(c_p + p) + \delta + \pi R\right), \textit{ or} \\
\left(\left(\pi\alpha c_p + \delta^2 < \alpha\delta\pi(c_p + p) + \delta + \pi R\right) \textit{ and } \left(\left(\frac{R}{\alpha(1-\delta)} \leq p\right) \textit{ or}\right.\right. \\
\left.\left.\left(\frac{R}{\alpha(1-\delta)} > p \textit{ and } \pi R^2(\alpha(\delta-1)(c_p+p)+R) \leq (\delta-1)\delta(\alpha(\delta-1)p+R)^2 \textit{ and } \frac{R}{\alpha - \alpha\delta} < c_p+p\right)\right)\right).
\end{gathered} \tag{A.29}$$

To solve for the thresholds v_r and v_p, using $u(\sigma) = v_p - v_{nr}$, we solve

$$v_r = \frac{p + R\pi u(\sigma)}{1 - \delta\pi\alpha u(\sigma)} = \frac{p + R\pi(v_p - v_r)}{1 - \delta\pi\alpha(v_p - v_r)}, \text{ and} \tag{A.30}$$

$$v_p = \frac{c_p - R\pi u(\sigma)}{\delta\pi\alpha u(\sigma)} = \frac{c_p - R\pi(v_p - v_r)}{\delta\pi\alpha(v_p - v_r)}. \tag{A.31}$$

Solving for v_p in terms of v_r in (A.30), we have

$$v_p = v_r + \frac{v_r - p}{(R + v_r\alpha\delta)\pi}. \tag{A.32}$$

Substituting this into (A.31), we have that v_r must be a zero of the cubic equation:

$$f_3(x) \triangleq \delta^2\alpha^2\pi x^3 - \alpha\delta(-1 - 2R\pi + (c_p + p)\delta\pi\alpha)x^2 + (R^2\pi - 2\alpha\delta(p + (c_p + p)R\pi))x + p^2\alpha\delta - (c_p + p)R^2\pi. \quad \text{(A.33)}$$

To find which root of the cubic v_r must be, note that the cubic's highest order term is $\delta^2\alpha^2\pi x^3 > 0$, so $\lim_{x\to-\infty} f_3(x) = -\infty$ and $\lim_{x\to\infty} f_3(x) = \infty$. Note $f_3(-\frac{R}{\alpha\delta}) = \alpha\delta\left(p + \frac{R}{\alpha\delta}\right)^2 > 0$, $f_3(p) = -c_p(R + p\alpha\delta)^2\pi < 0$, and $f_3(c_p + p) = c_p^2\alpha\delta > 0$. Then the root between p and $c_p + p$ is the largest positive root of the cubic. Since $v_r - p > 0$ in equilibrium, we have that v_r is uniquely defined as the largest root of the cubic, lying past p. Then using (A.32) to define v_p, we have v_p.

For this to be an equilibrium, the necessary and sufficient conditions are $0 < v_r < v_p < 1$ and no consumer strictly prefers to not pay the ransom over either (B, P) or (B, R).

First, note that $v_r > p$ implies both $v_r > 0$ and $v_p > v_r$, from (A.32).

For $v_p < 1$, using (A.32), this is equivalent to $\delta\pi\alpha v_r^2 + (1 + R\pi - \delta\pi\alpha)v_r - R\pi < p$. For this quadratic in v_r to be less than a constant, v_r needs to be between the two roots of the quadratic, $\frac{-1-R\pi+\delta\pi\alpha\pm\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}$. Both roots exist since the radicand is strictly positive.

Note that since $p \le 1$, then $\frac{-1-R\pi+\delta\pi\alpha-\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha} \le p$. Since we already have conditions for $v_r > p$, this implies that v_r is larger than the smaller root of the quadratic above.

Then the conditions we need for $v_p < 1$ are $v_r < \frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}$ and $\frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha} > p$. The latter condition is given in (B) of case (VI). With $\frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha} > p$, it follows that a necessary and sufficient for $v_r < \frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}$ is $f_3(\frac{-1-R\pi+\delta\pi\alpha+\sqrt{4\delta\pi\alpha(p+R\pi)+(1+R\pi-\delta\pi\alpha)^2}}{2\delta\pi\alpha}) > 0$. This is given in (A) of case (VI).

Lastly, we need to ensure that no consumer has an incentive to choose to not patch and not pay ransom. If $1 - \pi\alpha u(\sigma) \le 0$, then $v(1 - \pi\alpha u(\sigma)) \le 0$ for all v so that everyone would weakly prefer (NB, NP) over (B, NR). In this case, we do not need further conditions. Specifically, using (A.32), we have that $u(\sigma) = v_p - v_r = \frac{v_r - p}{(R + v_r\delta\alpha)\pi}$. So the condition that $1 - \pi\alpha u(\sigma) \le 0$ is equivalent to $v_r \ge \frac{R+p\alpha}{\alpha(1-\delta)}$. Since $\frac{R+p\alpha}{\alpha(1-\delta)} > p$, this is equivalent to $f_3(\frac{R+p\alpha}{\alpha(1-\delta)}) \le 0$, which boils down to $\pi\alpha c_p + \delta^2 \ge \alpha\delta\pi(c_p + p) + \delta + \pi R$.

On the other hand, if $\pi\alpha c_p + \delta^2 < \alpha\delta\pi(c_p + p) + \delta + \pi R$ so that $1 - \pi\alpha u(\sigma) > 0$, then a necessary and sufficient condition for no one to strictly prefer (B, NR) over the other options is for type $v = v_r$ to weakly prefer (NB, NP) over (B, NR). This would imply that all $v < v_r$ also have the same preference, from (A.4). Also, since $v = v_r$ is indifferent between (NB, NP) and (B, R), it follows that $v = v_r$ weakly prefers (B, R) over (B, NR). Then since only higher-valuation consumers would prefer paying ransom from (A.5), it follows that all

$v > v_r$ would also have the same preference. The condition that $v = v_r$ weakly prefers (NB, NP) over (B, NR) is $v_r \leq \frac{p}{1-\pi\alpha\left(\frac{v_r - p}{(R+v_r\alpha\delta)\pi}\right)}$. This simplifies to $v_r \geq \frac{R}{\alpha(1-\delta)}$.

Now if $\frac{R}{\alpha(1-\delta)} \leq p$, then no further conditions are needed since $v_r > p$ by definition of v_r. On the other hand, if $\frac{R}{\alpha(1-\delta)} > p$, then a necessary and sufficient condition for $v_r \geq \frac{R}{\alpha(1-\delta)}$ is for $f_3(\frac{R}{\alpha(1-\delta)}) \leq 0$ and $\frac{R}{\alpha(1-\delta)} < c_p$ (since $v_r < c_p$ by construction). This simplifies to $\pi R^2(\alpha(\delta - 1)(c_p + p) + R) \leq (\delta - 1)\delta(\alpha(\delta - 1)p + R)^2$ and $\frac{R}{\alpha - \alpha\delta} < c_p + p$. Altogether, these conditions above are summarized in (A.29). This concludes the proof of the consumer market equilibrium. ■

B Conditions for the Results in Section 4

Conditions for $0 < v_r < v_p < 1$ to arise in equilibrium: To facilitate reading of the paper, the conditions of Propositions 1-7 are listed here. For sufficiently small δ, condition set Γ_0 is defined to be the intersection of the conditions given below (and similarly for the other condition sets).

Condition set Γ_0:

(1) $\underline{c_p} < c_p < \overline{c_p}$

(2) $\underline{\alpha} < \alpha < \overline{\alpha}$

(3) $\underline{\underline{R}} < R < \underline{R}$

(4) $\underline{\pi} < \pi \leq 1$,

where

$$\underline{c_p} = \frac{1}{2}\left(3 - 2\sqrt{2}\right), \overline{c_p} = \frac{1}{2}\left(2 - \sqrt{2}\right), \underline{\alpha} = \frac{c_p(2-c_p)^2}{(1-c_p)^2}$$

$$\overline{\alpha} = \frac{-3 + \sqrt{(1-c_p)(9-25c_p)} + c_p(46 - 75c_p + 15\sqrt{(1-c_p)(9-25c_p)})}{16(1-3c_p)},$$

$$\underline{\underline{R}} = c_p\alpha, \underline{R} = \frac{\alpha}{2-c_p}, \text{ and } \underline{\pi} = \frac{c_p\alpha}{R^2 - c_p R\alpha} \text{ (which is positive).} \quad \text{(A.34)}$$

Condition set Γ_0 is a refinement on a set of sufficient conditions for $0 < v_r < v_p < 1$ to arise in equilibrium under optimal pricing. Specifically, Γ_0 is contained in the parameter region Γ_{0X} defined below.

Condition set Γ_{0X}:

(A) $R < \alpha$

(B) (i) $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$,

(ii) $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1 + c_p))\pi > 0$,

(iii) $\pi\alpha < c_p(8 + (2 + 3c_p)\pi\alpha)$

(iv) Either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or

$\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$

(C) $\alpha > \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$

(D) $c_pR + \alpha > 2R$

(E) $R^2\pi > c_p\alpha(1 + R\pi)$

(F) (i) $c_p(2 + R\pi) < R\pi$,

(ii) $\frac{R}{\alpha} < \frac{1+c_p}{2}$

The proof is similar to the proof of condition set Γ_{1X} except the difference is now that instead of finding conditions for the optimal profit of $0 < v_{nr} < v_r < v_p < 1$ to dominate $0 < v_r < v_p < 1$ and conditions for p^*_{IV} to be interior to the region defining $0 < v_{nr} < v_r < v_p < 1$, we now have conditions for the optimal profit of $0 < v_r < v_p < 1$ to dominate $0 < v_{nr} < v_r < v_p < 1$ and for the p^*_{VI} to be interior to the region defining $0 < v_r < v_p < 1$ (given in (D) and (F) above).

Conditions for $0 < v_{nr} < v_r < v_p < 1$ to arise in equilibrium: We characterize a region of the parameter space for which $0 < v_{nr} < v_r < v_p < 1$ arises under optimal pricing in equilibrium. For sufficiently small δ, condition set Γ_1 is defined to be the intersection of the conditions given below.

Condition set Γ_1:

(1) $\underline{c_p} < c_p < \overline{c_p}$

(2) $\underline{\alpha} < \alpha < \overline{\alpha}$

(3) $\underline{R} < R < \tilde{R}$

(4) $\underline{\pi} < \pi \leq 1$,

where all the bounds (except $\tilde{R}$) were defined in condition set Γ_0 and $\tilde{R} = \min\left(\frac{1}{2}\alpha(1 + c_p), \frac{1}{4}\left(\alpha + \sqrt{\alpha(16c_p + \alpha)}\right)\right)$.

Condition set Γ_1 is a refinement on a set of sufficient conditions for $0 < v_{nr} < v_r < v_p < 1$ to arise in equilibrium under optimal pricing. Specifically, Γ_1 is contained in the parameter region Γ_{1X} defined below.

Condition set Γ_{1X}:

In what follows, the conditions are categorized with letter labels that correspond to the labels in the Proof of Proposition 1.

(A) $R < \alpha$

(B) (i) $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$,

(ii) $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1 + c_p))\pi > 0$,

(iii) $\pi\alpha < c_p(8 + (2 + 3c_p)\pi\alpha)$

(iv) Either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or

$$\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$$

(C) $\alpha > \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$

(D) (i) $2c_p < R\pi$

(ii) $R(2R - \alpha)\pi < 2c_p\alpha$

(iii) $\alpha < 2R$

(iv) $R > c_p\alpha$

(E) $c_p < \frac{R^2\pi}{\alpha(1+R\pi)}$

(F) $\frac{R}{\alpha} > \frac{1}{2-c_p}$

Conditions for $0 < v_r < 1$ to arise in equilibrium: We characterize a region of the parameter space for which $0 < v_r < 1$ arises under optimal pricing in equilibrium. For sufficiently small δ, condition set Γ_2 is defined to be the intersection of the conditions given below.

Condition set Γ_2:

(1) $\underline{c_p} < c_p < \overline{c_p}$

(2) $\underline{\alpha} < \alpha < \overline{\alpha}$

(3) $\underline{R} < R < \tilde{R}$

(4) $\underline{\underline{\pi}} < \pi < \underline{\pi}$,

where all bounds (except $\underline{\underline{\pi}}$) were defined in condition set Γ_0 or Γ_1 and $\underline{\underline{\pi}} = \max\left(\frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}, \frac{2c_p}{\alpha}\right)$ (which is positive, given conditions (1), (2), and (3) above).

Condition set Γ_2 is a refinement on a set of sufficient conditions for $0 < v_r < 1$ to arise in equilibrium under optimal pricing. Specifically, Γ_2 is contained in the parameter region Γ_{2X} defined below.

Condition set Γ_{2X}:

(A) $R < \alpha$

(B) (i) $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$,

(ii) $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1 + c_p))\pi > 0$,

(iii) $\pi\alpha < c_p(8 + (2 + 3c_p)\pi\alpha)$

(iv) Either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or

$$\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$$

(C) $\alpha > \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$

(D) $R^2\pi < c_p\alpha(1 + R\pi)$

(E) (i) $2c_p(1 + R\pi) \geq R\pi$

(ii) Either $2 + 2R\pi > \pi\alpha$ and $2R(1 + R\pi - \pi\alpha) \leq \alpha$, or

$$2 + 2R\pi \leq \pi\alpha$$

(F) $\frac{R}{\alpha} > \frac{1}{2-c_p}$

The proof is similar to the proof of condition set Γ_{1X} except the difference is now that instead of finding conditions for the optimal profit of $0 < v_{nr} < v_r < v_p < 1$ to dominate $0 < v_r < 1$ and conditions for p^*_{IV} to be interior to the region defining $0 < v_{nr} < v_r < v_p < 1$, we now have conditions for the optimal profit of $0 < v_r < 1$ to dominate $0 < v_{nr} < v_r < v_p < 1$ and for the p^*_V to be interior to the region defining $0 < v_r < 1$ (given in (D) and (E) above). To facilitate the proof of Lemma A.3, we then impose the condition $\pi\alpha > 2c_p$, which becomes part of the lower bound on π. Under the conditions of this case, $\underline{\underline{\pi}} = \max\left(\frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}, \frac{2c_p}{\alpha}\right) < \underline{\pi} = \frac{c_p\alpha}{R^2-c_pR\alpha}$. However, note that $\pi\alpha > 2c_p$ is not a necessary condition for this case to arise.

Conditions for $0 < v_{nr} < v_r < 1$ to arise in equilibrium: We characterize a region of the parameter space for which $0 < v_{nr} < v_r < 1$ arises under optimal pricing in equilibrium. For sufficiently small δ, condition set Γ_3 is defined to be the intersection of the conditions given below.

Condition set Γ_3:

(1) $\underline{c_p} < c_p < \overline{c_p}$

(2) $\underline{\alpha} < \alpha < \overline{\alpha}$

(3) $\underline{R} < R < \tilde{R}$

(4) $0 < \pi < \frac{-2R+\alpha}{2R(R-\alpha)}$,

where all bounds were defined in condition set Γ_0 or Γ_1 and $\frac{-2R+\alpha}{2R(R-\alpha)} > 0$ under the conditions above.

Condition set Γ_3 is a refinement on a set of sufficient conditions for $0 < v_{nr} < v_r < 1$ to arise in equilibrium under optimal pricing. Specifically, Γ_3 is contained in the parameter region Γ_{3X} defined below.

Condition set Γ_{3X}:

(A) $R < \alpha$

(B) (i) $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$,

(ii) $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1 + c_p))\pi > 0$,

(iii) $\pi\alpha < c_p(8 + (2 + 3c_p)\pi\alpha)$

(iv) Either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or

$$\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$$

(C) $\alpha < \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$

(D) $R^2\pi < c_p\alpha(1 + R\pi)$

(E) (i) $2c_p(1 + R\pi) < R\pi$

(ii) $\pi < \frac{-2R+\alpha}{2R(R-\alpha)}$

(F) $\frac{R}{\alpha} > \frac{1}{2-c_p}$

As in the previous cases, the proof is similar to the proof of condition set Γ_{1X}.

C Proofs of Main Results

Proof of Proposition 1: From Lemma A.1, a unique consumer market equilibrium arises, given a price p. Within each region of the parameter space defined by Lemma A.1, the thresholds v_{nr}, v_r, and v_p are smooth functions of the parameters, as well as the vendor's price p. In the cases where the thresholds are given in closed-form, this is clear. In the cases where these thresholds are implicitly defined as the root of a polynomial, then the smoothness of the thresholds in the parameters follows from the Implicit Function Theorem. Specifically, for each of those cases, the threshold defined was the most positive root v_{nr}^* (or v_r^*) of a cubic function of v_{nr} (or v_r), $f(v_{nr}, p) = 0$. Moreover, the cubic $f(v_{nr}, p)$ has two local extrema in v_{nr} and is negative to the left of v_{nr}^* and positive to the right of it ($f(v_{nr}^* - \epsilon, p) < 0$ and $f(v_{nr}^* + \epsilon, p) > 0$ for arbitrarily small $\epsilon > 0$). Therefore, $\frac{\partial f}{\partial v_{nr}}(v_{nr}, p) \neq 0$ so that the Implicit Function Theorem applies. The thresholds being smooth in p implies that the profit function for each case of the parameter space defined by Lemma A.1 is smooth in p. In our analysis, we use asymptotic analysis to characterize the equilibrium prices and profits when needed, using Taylor Series representations in δ of the thresholds, price, and profit expressions. In the following paragraphs, we find the profit-maximizing interior solution within the compact closure of each subcase to characterize the conditions under which $0 < v_{nr} < v_r < v_p < 1$ arises in equilibrium under optimal pricing.

Sufficient conditions for $0 < v_{nr} < v_r < v_p < 1$ to arise in equilibrium are given in the following:

(A) the case $0 < v_{nr} < 1$ does not arise in a consumer market equilibrium,

(B) the vendor's interior maximizing price of $0 < v_{nr} < v_p < 1$ does not induce $0 < v_{nr} < v_p < 1$ in a consumer market equilibrium since that price lies outside of the region which would induce $0 < v_{nr} < v_p < 1$ in equilibrium,

(C) the vendor's interior maximizing price of $0 < v_{nr} < v_r < 1$ does not induce $0 < v_{nr} < v_r < 1$ in a consumer market equilibrium since that price lies outside of the region which would induce $0 < v_{nr} < v_r < 1$ in equilibrium,

(D) the vendor's interior maximizing price of $0 < v_{nr} < v_r < v_p < 1$ indeed induces $0 < v_{nr} < v_r < v_p < 1$ in a consumer market equilibrium,

(E) the vendor's interior maximal profit of $0 < v_{nr} < v_r < v_p < 1$ dominates his interior maximal profit of $0 < v_r < 1$, and

(F) the vendor's interior maximal profit of $0 < v_{nr} < v_r < v_p < 1$ dominates his interior maximal profit of $0 < v_{nr} < 1$.

(A): For (A), note that region of the parameter space defining $0 < v_{nr} < 1$ is given in part (I) of Lemma A.1. In particular, $R \geq \alpha(1-\delta)$ is a necessary condition for $0 < v_{nr} < 1$ to arise. If $R < \alpha$, then there would exist some $\tilde{\delta} > 0$ such that for $\delta < \tilde{\delta}$, $R \geq \alpha(1-\delta)$ is never satisfied. Therefore, a sufficient condition to guarantee that $0 < v_{nr} < 1$ does not arise in equilibrium is $R < \alpha$.

To sum up:

Summary of Conditions for (A)

(i) $R < \alpha$

(B): For (B), first suppose that $0 < v_{nr} < v_p < 1$ is induced. From (A.13), we have that v_{nr} is the largest root of the cubic:

$$f_1(x) \triangleq \pi\alpha x^3 + (1 - \pi\alpha(c_p + p))x^2 - 2px + p^2. \tag{A.35}$$

Then in equilibrium, p^*_{II} and v_{nr} must solve $\pi\alpha v_{nr}^3 + (1-\pi\alpha(c_p+p))v_{nr}^2 - 2pv_{nr} + p^2 = 0$. From this, we have that

$$p^*_{II} = \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} \pm \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)}\right). \tag{A.36}$$

Can $p^*_{II} = \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} + \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)}\right)$? Suppose it were. Then it follows that $p^*_{II} > \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} + \pi\alpha v_{nr}\right) = v_{nr}(1 + \pi\alpha v_{nr})$. This is a contradiction, since $v_{nr} > p^*_{II}$ in equilibrium (otherwise, some purchasing consumers would derive negative utility upon purchasing). Therefore, we have in equilibrium that

$$p^*_{II} = \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)}\right). \tag{A.37}$$

At the same time, implicitly differentiating (A.13) to find $v'_{nr}(p)$, we have that

$$v'_{nr}(p) = \frac{2p - v_{nr}(p)(2 + \pi\alpha v_{nr}(p))}{2p + v_{nr}(p)(-2 + 2(c_p+p)\pi\alpha - 3\pi\alpha v_{nr}(p))}. \tag{A.38}$$

The profit function in this case is $\Pi_{II}(p) = p(1 - v_{nr}(p))$. Let C_{II} be the compact closure of the region of the parameter space defining $0 < v_{nr} < v_p < 1$, given in part (II) of Lemma A.1. By the Weierstrass extreme value theorem, there exists p in C_{II} that maximizes $\Pi_{II}(p)$. If this p is interior to C_{II}, this unconstrained maximizer satisfies the first-order condition. Then p^*_{II} must satisfy $1 - v_{nr}(p) - pv'_{nr}(p) = 0$. Substituting in (A.38), we have that p^*_{II} and v_{nr} solve

$$1 - v_{nr} - p\left(\frac{2p - v_{nr}(2 + \pi\alpha v_{nr})}{2p + v_{nr}(-2 + 2(c_p+p)\pi\alpha - 3\pi\alpha v_{nr})}\right) = 0. \tag{A.39}$$

Solving for p, we have that

$$p^*_{II} = \frac{1}{4}\Bigg(2 + 2\pi\alpha v_{nr} - \pi\alpha v_{nr}^2 \pm \sqrt{8(-1+v_{nr})v_{nr}(2 - c_p\pi\alpha + 3v_{nr}\pi\alpha) + (-2 + (-2+v_{nr})v_{nr}\pi\alpha)^2}\Bigg). \tag{A.40}$$

To rule out p^*_{II} being the larger root (denote it $\tilde{p}_{II}$), it is sufficient to have the condition $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$. Suppose that p^*_{II} was the larger root. We will show that it would then be larger than v_{nr}, which cannot happen in equilibrium. The inequality $\tilde{p}_{II} > v_{nr}$ is equivalent to $\sqrt{8(-1+v_{nr})v_{nr}(2 - c_p\pi\alpha + 3v_{nr}\pi\alpha) + (-2 + (-2+v_{nr})v_{nr}\pi\alpha)^2} > -2 + v_{nr}(4 + (-2 + v_{nr})\pi\alpha)$. But $-2 + v_{nr}(4 + (-2+v_{nr})\pi\alpha) < 0$ from $0 < v_{nr} < 1$ and $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$.

Therefore, $\tilde{p}_{II} > v_{nr}$, which cannot happen in equilibrium. So we have in equilibrium that

$$p^*_{II} = \frac{1}{4}\Bigg(2 + 2\pi\alpha v_{nr} - \pi\alpha v_{nr}^2 - \sqrt{8(-1+v_{nr})v_{nr}(2 - c_p\pi\alpha + 3v_{nr}\pi\alpha) + (-2 + (-2+v_{nr})v_{nr}\pi\alpha)^2}\Bigg). \quad \text{(A.41)}$$

We want to show that $p^*_{II} > \frac{1-c_p}{2}$. To do that, we will show the following. First, we will show that the two curves defined by (A.37) and (A.41) intersect at a point (p, v_{nr}) in which both $v_{nr} > 0$ and $p > \frac{1-c_p}{2}$.

First, define $p_1(v_{nr})$ and $p_2(v_{nr})$ to be the expressions given in (A.37) and (A.41), respectively. Then $p_2(v_{nr}) = \frac{1-c_p}{2}$ for three possible solutions $v_{nr} = \frac{1+c_p}{2}, \frac{-1+\pi\alpha\pm\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}$. Note that $\frac{-1+\pi\alpha-\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha} < 0$ and $\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha} > 0$, so only $v_{nr} = \frac{1+c_p}{2}$ and $v_{nr} = \frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}$ are positive solutions in v_{nr} to the equation $p_2(v_{nr}) = \frac{1-c_p}{2}$.

Then $p_1(\frac{1+c_p}{2}) = \frac{1}{8}(1+c_p)\left(4 + (1+c_p)\pi\alpha - \sqrt{\pi\alpha(16c_p + \pi\alpha(1+c_p)^2)}\right) > p_2(\frac{1+c_p}{2}) = \frac{1-c_p}{2}$, from $c_p \in (0,1)$ and $\pi\alpha > 0$.

Next, we will find conditions such that $\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha} < \frac{1+c_p}{2}$ and conditions such that $p_1(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}) < \frac{1-c_p}{2} = p_2(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha})$. Then we would have $p_1(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}) < p_2(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha})$, $p_1(\frac{1+c_p}{2}) > p_2(\frac{1+c_p}{2})$, and $\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha} < \frac{1+c_p}{2}$ Applying the Intermediate Value Theorem, this implies that there would exist an intersection of $p_1(v_{nr})$ and $p_2(v_{nr})$ between $(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}$ and $\frac{1+c_p}{2}$, such that $p > \frac{1-c_p}{2}$.

The condition $\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha} < \frac{1+c_p}{2}$ can be simplified as $\pi\alpha < c_p(8+(2+3c_p)\pi\alpha)$.

The condition $p_1(\frac{-1+\pi\alpha+\sqrt{1+\pi\alpha(1-3c_p+\pi\alpha)}}{3\pi\alpha}) < \frac{1-c_p}{2}$ can be simplified as: either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or $\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$.

Then under these combined conditions, we have that $p^*_{II} > \frac{1-c_p}{2}$.

Now, we want to find conditions such that this optimal price will not induce $0 < v_{nr} < v_p < 1$ in equilibrium. Note that region of the parameter space defining $0 < v_{nr} < v_p < 1$ is given in part (II) of Lemma A.1. To make sure $0 < v_{nr} < v_p < 1$ does not arise in equilibrium for sufficiently small δ, it suffices to violate the first condition in part (II) of Lemma A.1 by having $c_p\alpha(R - c_p\alpha(1-\delta))(1-\delta)^2 > R^2(R - \alpha(c_p+p)(1-\delta))\pi$ at $p = p^*_{II}$. It suffices to have $c_p\alpha(R - c_p\alpha) + R^2(-R + (c_p+p)\alpha)\pi > 0$ at the $p = p^*_{II}$. This simplifies to $p^*_{II} > \frac{(R-c_p\alpha)(-c_p\alpha+R^2\pi)}{R^2\pi\alpha}$. But from the above, we found conditions for $p^*_{II} > \frac{1-c_p}{2}$, so a sufficient condition for $p^*_{II} > \frac{(R-c_p\alpha)(-c_p\alpha+R^2\pi)}{R^2\pi\alpha}$ is $\frac{1-c_p}{2} > \frac{(R-c_p\alpha)(-c_p\alpha+R^2\pi)}{R^2\pi\alpha}$. This simplifies to $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1+c_p))\pi > 0$.

Summary of Conditions for (B)

(i) $c_p > \frac{1}{2}\left(3 - 2\sqrt{2}\right)$,

(ii) $2c_p\alpha(R - c_p\alpha) + R^2(-2R + \alpha(1 + c_p))\pi > 0$,

(iii) $\pi\alpha < c_p(8 + (2 + 3c_p)\pi\alpha)$,

(iv) Either $2 + \pi\alpha(7 - 15c_p + 6\pi\alpha) < 0$ or

$$\pi\alpha(3 + 75c_p^2 + 8\pi\alpha) < 2c_p(8 + \pi\alpha(23 + 12\pi\alpha))$$

(C): For (C), first suppose that $0 < v_{nr} < v_r < 1$ is induced. From (A.19), we have

$$v_{nr} = \frac{-1 + \pi\alpha + \sqrt{1 + \pi\alpha(-2 + 4p + \pi\alpha)}}{2\pi\alpha}. \tag{A.42}$$

The profit function in this case is $\Pi_{III}(p) = p(1 - v_{nr}(p))$. Let C_{III} be the compact closure of the region of the parameter space defining $0 < v_{nr} < v_r < 1$, given in part (III) of Lemma A.1. By the Weierstrass extreme value theorem, there exists p in C_{III} that maximizes $\Pi_{III}(p)$. If this p is interior to C_{III}, the unconstrained maximizer satisfies the first-order condition.

Differentiating the profit function with respect to p and solving for the positive root of the quadratic, we have that

$$p_{III}^* = \frac{1}{9}\left(4 - \frac{1}{\pi\alpha} - \pi\alpha + \frac{\sqrt{1 + \pi\alpha + (\pi\alpha)^3 + (\pi\alpha)^4}}{\pi\alpha}\right). \tag{A.43}$$

The second-order condition is satisfied at this interior solution. The region of the parameter space defining $0 < v_{nr} < v_r < 1$ is given in part (III) of Lemma A.1, so for this price to actually induce $0 < v_{nr} < 1$, p_{III}^* must satisfy those conditions.

In particular, a sufficient condition so that p_{III}^* lies outside of the conditions specified in (III) of Lemma A.1 is for $-2R\pi + (1 - \delta)(-1 + \pi\alpha + \sqrt{1 + \pi\alpha(-2 + 4p + \pi\alpha)}) > 0$ at $p = p_{III}^*$ for sufficiently high δ. Omitting the algebra, this condition is equivalent to $\alpha > \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$.

Summary of Conditions for (C)

(i) $\alpha > \frac{1+4R\pi-\sqrt{1+4R^2\pi^2}}{2\pi}$

(D): For (D), first suppose that $0 < v_{nr} < v_r < v_p < 1$ is induced. From (A.25), we have that v_{nr} is the largest root of

$$\delta\pi\alpha x^3 + (\delta + \pi(R - \alpha(c_p + \delta p)))x^2 - p(2\delta + R\pi)x + p^2\delta = 0. \tag{A.44}$$

A generalization of the Implicit Function Theorem gives that v_{nr} is not only a smooth function of the parameters, but it is also an analytic function of the parameters so that it can be represented locally as a Taylor series of its parameters. More specifically, since $f_2'(x) \neq 0$ at the root for which v_{nr} is defined, there exists a $\delta_1 > 0$ such that for $\delta < \delta_1$, $v_{nr} = \sum_{k=0}^{\infty} a_k \delta^k$ for some sequence of coefficients α_k. Substituting $v_{nr} = \sum_{k=0}^{\infty} a_k \delta^k$ into (A.44), we have that $-a_0(a_0\pi\alpha c_p - a_0 R\pi + pR\pi) + \sum_{k=1}^{\infty} a_k \delta^k = 0.$

Then $a_0 = 0$ or $a_0 = \frac{pR}{R-\alpha c_p}$ are the only solutions for a_0 that make the first term zero. Now, $a_0 \neq 0$, since otherwise $v_{nr} < p$ for sufficiently low δ, which cannot happen. So $a_0 = \frac{pR}{R-\alpha c_p}$. Then substituting $v_{nr} = \frac{pR}{R-\alpha c_p} + \sum_{k=1}^{\infty} a_k \delta^k$ into (A.44) and similarly solving for a_1, we have $a_1 = \frac{c_p p \alpha^2 (c_p^2 \pi\alpha - R\pi(c_p + pR\pi))}{R(R-c_p\alpha)^3\pi^2}$. Continuing on this way, we have that

$$v_{nr} = \frac{pR}{R - c_p\alpha} + \frac{c_p p\alpha^2(-c_p R + c_p^2\alpha - pR^2\pi)\delta}{R(R-c_p\alpha)^3\pi} - \frac{c_p p\alpha^3(-c_p R + c_p^2\alpha - pR^2\pi)(c_p(-2R + c_p\alpha)(-R + c_p\alpha) + pR^2(R + c_p\alpha)\pi)\delta^2}{R^3(R-c_p\alpha)^5\pi^2} + \sum_{k=3}^{\infty} a_k\delta^k. \tag{A.45}$$

The profit function in this case is $\Pi_{IV}(p) = p(1 - v_{nr}(p))$. Let C_{IV} be the compact closure of the region of the parameter space defining $0 < v_{nr} < v_r < v_p < 1$, given in part (IV) of Lemma A.1. By the Weierstrass extreme value theorem, there exists p in C_{IV} that maximizes $\Pi_{IV}(p)$. If this p is interior to C_{IV}, the unconstrained maximizer satisfies the first-order condition.

Substituting (A.45) into the first-order condition and expanding the optimal price as a Taylor series in δ, we can then characterize the asymptotic expansion of the optimal price in the same way we had done above with v_{nr}. Omitting the algebra, we have that the interior solution is given by

$$p_{IV}^* = \frac{R - c_p\alpha}{2R} + \frac{c_p\alpha^2(4c_p + 3R\pi)\delta}{8R^3\pi} + \frac{c_p\alpha^3(16c_p^3\alpha - 4R^3\pi^2 + c_p R^2\pi(-18 + 5\pi\alpha) + 2c_p^2 R(-8 + 9\pi\alpha))\delta^2}{16R^5(R - c_p\alpha)\pi^2} + \sum_{k=3}^{\infty} a_k\delta^k. \tag{A.46}$$

The profit associated with this price is given by

$$\Pi_{IV}^* = \frac{R - c_p\alpha}{4R} + \frac{c_p\alpha^2(2c_p + R\pi)\delta}{8R^3\pi} + \frac{c_p\alpha^3(32c_p^3\alpha - 4R^3\pi^2 + 8c_p^2R(-4 + 3\pi\alpha) + c_pR^2\pi(-24 + 5\pi\alpha))\delta^2}{64R^5(R - c_p\alpha)\pi^2} + \sum_{k=3}^{\infty} a_k\delta^k. \quad \text{(A.47)}$$

The consumer market equilibrium conditions for $0 < v_{nr} < v_r < v_p < 1$ are given in part (IV) of Lemma A.1. We want these conditions to hold for $p = p_{IV}^*$ for sufficiently small δ. Just for reference, the conditions are given again below.

(a) $R > p\alpha(1 - \delta)$

(b) $R^2(R - (c_p + p)\alpha(1 - \delta))\pi > -(R - p\alpha(1 - \delta))^2\delta(1 - \delta)$

(c) $\alpha(c_p - \delta) + \alpha^2\pi(c_p(\pi\alpha + 2p - 2) + \delta(\alpha(p-1)\pi - 3p + 2)) + \sqrt{\pi\alpha(\pi\alpha + 4p - 2) + 1}(\alpha(\pi\alpha(c_p + \delta(p-1)) - c_p + \delta) + \alpha(p-1)\pi R + R) + R(\pi\alpha(\alpha(p - 1)\pi - 3p + 2) - 1) < 0$

(d) Either $\Big(\big(\alpha c_p(\delta - 1)^3(2\pi\alpha p + 1) + \sqrt{(\delta-1)^2 + 2(\delta - 1)\pi(2\alpha(\delta - 1)p + R) + \pi^2R^2} \times (\alpha c_p(\delta - 1)^2 + (\delta - 1)R(\pi\alpha(c_p + p) + 1) + \pi R^2) + (\delta - 1)\pi R^2(\pi\alpha(c_p + p) + 2) + (\delta - 1)^2R(2\pi\alpha c_p + 3\pi\alpha p + 1) + \pi^2R^3 < 0\big)$ and $\big(\delta + \sqrt{4\alpha(\delta - 1)^2p\pi + (\delta + \pi R - 1)^2} < \pi R + 1\big)\Big)$, or

$\Big(\big(\alpha c_p(\delta - 1)^3(2\pi\alpha p + 1) + \sqrt{(\delta-1)^2 + 2(\delta - 1)\pi(2\alpha(\delta - 1)p + R) + \pi^2R^2} \times (-\alpha c_p(\delta - 1)^2 - (\delta - 1)R(\pi\alpha(c_p + p) + 1) - \pi R^2) + (\delta - 1)\pi R^2(\pi\alpha(c_p + p) + 2) + (\delta - 1)^2R(2\pi\alpha c_p + 3\pi\alpha p + 1) + \pi^2R^3 > 0\big)$ and $\big(\pi(R - 2\alpha(1 - \delta)p) > -\delta + \sqrt{4\alpha(\delta - 1)^2p\pi + (\delta + \pi R - 1)^2} + 1\big)\Big)$

Substituting in (A.46) for each of the conditions in the list above, we derive the sufficient conditions for these conditions to hold for $p = p_{IV}^*$ for sufficiently small δ. Note that some of these conditions are redundant given the others, and they can be simplified to the following set of conditions:

Summary of Conditions for (D)

(i) $2c_p < R\pi$

(ii) $R(2R - \alpha)\pi < 2c_p\alpha$

(iii) $\alpha < 2R$

(iv) $R > c_p\alpha$

(E): For (E), from (A.28), we have that

$$v_r = \frac{-1 - R\pi + \delta\pi\alpha + \sqrt{4\delta\pi\alpha(p + R\pi) + (1 + R\pi - \delta\pi\alpha)^2}}{2\delta\pi\alpha}. \tag{A.48}$$

The vendor's profit function is given as $\Pi_V(p) = p(1 - v_r(p))$. Solving for the first-order condition (and looking at only the positive root) gives

$$p_V^* = \Bigg(-1 - 2R\pi + 4\delta\pi\alpha - R^2\pi^2 - 2R\delta\alpha\pi^2 - (\delta\pi\alpha)^2 + (1 + R\pi + \delta\pi\alpha)\sqrt{1 + \pi(2R - \delta\alpha + (R + \delta\alpha)^2\pi)}\Bigg)\Bigg(9\delta\pi\alpha\Bigg)^{-1}. \tag{A.49}$$

Substituting (A.49) into the profit function of this case yields the associated maximal profit of this case. Characterizing this profit expression in terms of a Taylor Series expansion, we have that

$$\Pi_V^* = \frac{1}{4(1 + R\pi)} + \sum_{k=1}^{\infty} a_k\delta^k. \tag{A.50}$$

On the other hand, the vendor's profit of $0 < v_{nr} < v_r < v_p < 1$ was given in (A.47). Comparing the two, there exists $\delta_1 > 0$ such that for $\delta < \delta_1$, it follows that $\Pi_{IV}^* > \Pi_V^*$ whenever $c_p < \frac{R^2\pi}{\alpha(1+R\pi)}$. To sum up:

Summary of Conditions for (E)

(i) $c_p < \frac{R^2\pi}{\alpha(1+R\pi)}$

(F): For (F) (in which $0 < v_r < v_p < 1$ arises in equilibrium), from (A.33), we have that v_r is the largest root of

$$\delta^2\alpha^2\pi x^3 - \alpha\delta(-1 - 2R\pi + (c_p + p)\delta\pi\alpha)x^2 + (R^2\pi - 2\alpha\delta(p + (c_p + p)R\pi))x + p^2\alpha\delta - (c_p + p)R^2\pi = 0. \tag{A.51}$$

Characterizing the asymptotic expansion of v_r as we had done for earlier cases, we have that

$$v_r = c_p + p - \frac{c_p^2\alpha\delta}{R^2\pi} + \frac{2(c_p^3\alpha^2 + c_p^3R\alpha^2\pi + c_p^2pR\alpha^2\pi)\delta}{R^4\pi^2} + \sum_{k=3}^{\infty} a_k\delta^k. \tag{A.52}$$

The profit function for this case is given by $\Pi_{VI} = p(1 - v_r(p))$. Assuming an interior solution, the first-order condition gives

$$p^*_{VI} = \frac{1 - c_p}{2} + \frac{c_p^2 \alpha \delta}{2R^2 \pi} + \sum_{k=2}^{\infty} a_k \delta^k. \tag{A.53}$$

The corresponding profit is given by

$$\Pi^*_{VI} = \frac{1}{4}(1 - c_p)^2 + \frac{(1 - c_p) c_p^2 \alpha \delta}{2R^2 \pi} + \sum_{k=2}^{\infty} a_k \delta^k. \tag{A.54}$$

The vendor's profit of $0 < v_{nr} < v_r < v_p < 1$ was given in (A.47). Comparing this with (A.54), there exists $\delta_2 > 0$ such that for $\delta < \delta_2$, it follows that $\Pi^*_{IV} > \Pi^*_{VI}$ whenever $\frac{R}{\alpha} > \frac{1}{2 - c_p}$. To sum up:

Summary of Conditions for (F)

(i) $\frac{R}{\alpha} > \frac{1}{2 - c_p}$

The intersection of the conditions across (A), (B), . . . , (F) gives the conditions for Proposition 1 in the condition set Γ_{1X}.

We consolidate the conditions in Γ_{1X} and add additional simplifying assumptions to reduce the complexity of the characterization of the region of interest, yielding condition set Γ_1. Consequently, $0 < v_{nr} < v_r < v_p < 1$ arises in equilibrium under optimal pricing in condition set Γ_1 for sufficiently low δ. Following the same approach above, we $0 < v_r < v_p < 1$ arises in equilibrium under optimal pricing in condition set Γ_0 and $0 < v_r < 1$ arises in equilibrium in condition set Γ_2 for sufficiently low δ.

We want to show that in condition set Γ_0, the vendor's profit under optimal pricing decreases in R, and the vendor's price also decreases in R while the market shrinks. From (A.54), $\frac{d}{dR}\Pi^*_{VI} < 0$ for sufficiently small δ. From (A.53), $\frac{d}{dR}p^*_{VI} < 0$ for sufficiently small δ as well. Substituting (A.53) into (A.52) and differentiating with respect to R, we see that $\frac{d}{dR}v^*_r > 0$. Therefore, the total market size $1 - v^*_r$ decreases in R.

We want to show that in condition set Γ_1, the vendor's profit under optimal pricing increases in R, and the vendor's price also increases in R while the market expands. From (A.47), $\frac{d}{dR}\Pi^*_{IV} > 0$ for sufficiently small δ. From (A.46), $\frac{d}{dR}p^*_{IV} > 0$ for sufficiently small δ as well. Substituting (A.46) into (A.45) and differentiating with respect to R, we see that $\frac{d}{dR}v^*_{nr} < 0$. Therefore, the total market size $1 - v^*_{nr}$ increases in R. ■

Lemma A.2. *The complete threshold characterization of the consumer market equilibrium of the benchmark case is as follows:*

(I) $(0 < v_{nr} < 1)$, *where* $v_{nr} = \frac{\pi\alpha - 1 + \sqrt{1 + \pi\alpha(-2 + 4p + \pi\alpha)}}{2\pi\alpha}$*:*

(A) $p < 1$

(B) *Either* $1 + \pi\alpha - 2c_p < 0$*, or*

(C) $1 + \pi\alpha - 2c_p \geq 0$ *and* $p \geq \frac{(1-c_p)(\pi\alpha - c_p)}{\pi\alpha}$

(II) $(0 < v_{nr} < v_p < 1)$*, where* v_{nr} *is the largest root of the cubic* $f(x) = \pi\alpha x^3 + (1-(c_p + p)\pi\alpha)x^2 - 2px + p^2$ *and* $v_p = \frac{c_p v_{nr}}{v_{nr} - p}$*:*

(A) $p < 1$

(B) $c_p + (-1 + c_p + p)\pi\alpha < c_p^2$

Proof of Lemma A.2: This follows from the proof of Lemma A.1 with $\delta = 1$. ■

Lemma A.3. *Under condition sets* Γ_0*,* Γ_1*, or* Γ_2*, if* $0 < v_{nr} < v_p < 1$ *arises in equilibrium under optimal pricing of the benchmark case, then* $v_{nr} > \frac{1}{2}$*.*

Proof of Lemma A.3: From (A.37), we have that an expression of the vendor's price as a function of v_{nr} when $0 < v_{nr} < v_p < 1$ is induced in equilibrium is given by

$$p_{II}^* = \frac{1}{2} v_{nr} \left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)} \right). \tag{A.55}$$

The conditions of either Γ_0, Γ_1, or Γ_2 imply that $p_{II}^* > \frac{1-c_p}{2}$, from sub-conditions (B) in Γ_{0X}, Γ_{1X}, or Γ_{2X} respectively. Then $\frac{1}{2} v_{nr} \left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)} \right) > \frac{1-c_p}{2}$. Simplifying, this is equivalent to $2 + \pi\alpha v_{nr} - \frac{1-c_p}{v_{nr}} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)} > 0$. In equilibrium, v_{nr} must satisfy this inequality.

Define $q(x) = 2 + \pi\alpha x - \frac{1-c_p}{x} - \sqrt{\pi\alpha(4c_p + \pi\alpha x^2)}$. Note that the equilibrium v_{nr} needs to satisfy $q(v_{nr}) > 0$. We note $\frac{dq}{dx} > 0$ under $0 < x < 1$, $\pi\alpha > 0$, and $0 < c_p < 1$. Moreover, $\lim_{x \to 0^+} <= -\infty$ and $\lim_{x \to 1^-} > 0$. Let $\tilde{v}_{nr}$ denote the unique root of $q(x)$ between 0 and 1. Then $\tilde{v}_{nr}$ is a lower bound on the equilibrium v_{nr}. In particular, a necessary and sufficient condition for $\tilde{v}_{nr} > \frac{1}{2}$ is for $q(\frac{1}{2}) < 0$. This is equivalent to $\pi\alpha > 2c_p$.

We will need to show this condition holds for Γ_0, Γ_1, and Γ_2.

For Γ_0, given any parameters c_p, α, and R, $\pi\alpha$ is smallest when π is the lowest it can be given those parameters, which is the lower bound on π. So $\pi\alpha > \left(\frac{c_p \alpha}{R^2 - c_p R \alpha} \right) \alpha$. This bound is strictly decreasing in R, given the conditions of Γ_0, so $\pi\alpha > \left(\frac{c_p \alpha}{R^2 - c_p R \alpha} \right) \alpha |_{R = \frac{\alpha}{2 - c_p}}$, where $R = \frac{\alpha}{2 - c_p}$ is the upper bound on R under Γ_0, given c_p and α. So $\pi\alpha > \frac{c_p(2-c_p)^2}{(1-c_p)^2}$. Under Γ_0 (specifically, when $\frac{1}{2}\left(3 - 2\sqrt{2}\right) < c_p < \frac{1}{2}\left(2 - \sqrt{2}\right)$), we have $\frac{c_p(2-c_p)^2}{(1-c_p)^2} > 2c_p$. Therefore, $\pi\alpha > 2c_p$ under Γ_0.

For Γ_1, given any parameters c_p, α, and R, $\pi\alpha$ is smallest when π is the lowest it can be given those parameters, which is the lower bound on π. Again, this implies $\pi\alpha >$

$\left(\frac{c_p\alpha}{R^2-c_pR\alpha}\right)\alpha$. This bound is strictly decreasing in R, given the conditions of Γ_1, so $\pi\alpha > \left(\frac{c_p\alpha}{R^2-c_pR\alpha}\right)\alpha|_{R=\overline{R}}$, where $\overline{R} = \min\left(\frac{1}{2}\alpha(1+c_p), \frac{1}{4}\left(\alpha + \sqrt{\alpha(16c_p+\alpha)}\right)\right)$ is the upper bound on R under Γ_1, given c_p and α. Since this bound is strictly decreasing in R, it follows that $\left(\frac{c_p\alpha}{R^2-c_pR\alpha}\right)\alpha|_{R=\overline{R}} \geq \left(\frac{c_p\alpha}{R^2-c_pR\alpha}\right)\alpha|_{R=\frac{1}{2}\alpha(1+c_p)} = \frac{4c_p}{1-c_p^2}$. Therefore, $\pi\alpha > \frac{4c_p}{1-c_p^2}$. Under Γ_1 (specifically, when $\frac{1}{2}\left(3-2\sqrt{2}\right) < c_p < \frac{1}{2}\left(2-\sqrt{2}\right)$), we have $\frac{4c_p}{1-c_p^2} > 2c_p$. Therefore, $\pi\alpha > 2c_p$ under Γ_1.

For Γ_2, this condition follows from $\pi > \underline{\underline{\pi}}$. This bound is strictly decreasing in R, given the conditions of Γ_1. So $\pi\alpha > \left(\frac{c_p\alpha}{R^2-c_pR\alpha}\right)\alpha|_{R=\frac{\alpha}{2-c_p}}$, where $R = \frac{\alpha}{2-c_p}$ is the upper bound on R under Γ_0, given c_p and α.

Then in all cases, we have $\pi\alpha > 2c_p$, which implies that $q(\frac{1}{2}) < 0$. Then it follows that $v_{nr} > \tilde{v}_{nr} > \frac{1}{2}$. ■

Lemma A.4. *Under either condition set Γ_0, Γ_1, or Γ_2, if $0 < v_{nr} < v_p < 1$ arises in equilibrium under optimal pricing of the benchmark case, then $v_{nr} < \frac{1+c_p}{2}$.*

Proof of Lemma A.4: Again, from (A.37), we have that an expression of the vendor's optimal price as a function of v_{nr} when $0 < v_{nr} < v_p < 1$ is induced in equilibrium is given by

$$p^*_{II} = \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)}\right). \tag{A.56}$$

The vendor's profit function as a function of v_{nr} is given by $\Pi_{II}(v_{nr}) = p(v_{nr})(1-v_{nr})$. To prove the lemma, we will show that $\frac{d}{v_{nr}}\Pi_{II}(v_{nr}) < 0$ for $v_{nr} \in [\frac{1+c_p}{2}, 1]$.

Using the above expression of $p^*(v_{nr})$, we have that

$$\frac{d}{v_{nr}}\Pi_{II}(v_{nr}) = \frac{1}{2}\Bigg((-1+v_{nr})\pi\alpha v_{nr}\left(-1 + v_{nr}\sqrt{\frac{\pi\alpha}{4c_p+\pi\alpha v_{nr}^2}}\right) + (-1+2v_{nr})\left(-2+\pi\alpha v_{nr} + \sqrt{\pi\alpha(4c_p+\pi\alpha v_{nr}^2)}\right)\Bigg). \tag{A.57}$$

Now to show that this negative for all $v_{nr} \in [\frac{1+c_p}{2}, 1]$, we will show that it is negative when v_{nr} is a convex combination of $\frac{1+c_p}{2}$ and 1. In particular, substituting $v_{nr} = w + (1-w)\frac{1+c_p}{2}$ into (A.57), we will show that this expression is negative for all $w \in [0,1]$.

In particular, $\frac{d}{v_{nr}}\Pi_{II}(v_{nr})\big|_{v_{nr}=w+(1-w)\frac{1+c_p}{2}} < 0$ is equivalent to

$$\pi\alpha(32c_p(w+c_p-wc_p) - (1+3w(-1+c_p)-3c_p)(1+w+c_p-wc_p)^2\pi\alpha) < \sqrt{\pi\alpha\left(16c_p + (1+w+c_p-wc_p)^2\pi\alpha\right)}\Big(8w(1-c_p) + 8c_p - \pi\alpha + (-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha\Big). \tag{A.58}$$

Now we examine several subcases.

Subcase 1: $w \geq \frac{1}{3}$. First, suppose that $w \geq \frac{1}{3}$. This implies that > 0 and $\Big(8w(1-c_p)+ 8c_p - \pi\alpha + (-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha\Big) > 0$ as well, for any $\pi\alpha > 0$ and $0 < c_p < 1$. In this case, both the left and right side of the inequailty are positive. We isolate the radicand and square both sides. Simplifying and omitting the algebra, this is equivalent to

$$\begin{aligned} 64c_p(w+c_p-wc_p)^2 > -(4(w(-1+c_p)-c_p)(w^3(-1+c_p)^3+(-3+c_p)c_p(-1+3c_p)+ \\ w(-1+c_p)(1+c_p)(1+11c_p)-w^2(1-c_p)^2(2+15c_p))\pi\alpha+ \\ w(1+3w(-1+c_p)-3c_p)(-1+c_p)(1+w+c_p-wc_p)^3(\pi\alpha)^2) \end{aligned} \tag{A.59}$$

Now viewing the left-hand side as a constant function in α and the right-hand side as a quadratic function in α, we want to show that the quadratic in α is smaller than a constant in α. With $w \geq \frac{1}{3}$, the coefficient on α^2 on the right-hand side is negative. Then it suffices to show that the maximum of that quadratic is less than $64c_p(w+c_p-wc_p)^2$. Differentiating the right-hand side of the inequality with respect to α and solving for the maximum, we find that the maximizing α is negative. Therefore, the right-hand side of (A.59) is maximized at $\alpha = 0$, which would the right-hand side of the inequailty 0. Then to show that the inequality above holds for all $\alpha > 0$, it suffices to show that $64c_p(w+c_p-wc_p)^2 > 0$, which is true since $c_p > 0$ and $\frac{1}{3} \leq w \leq 1$. $\square$

Subcase 2a: $0 \leq w < \frac{1}{3}$ and $1 - \frac{2}{3(1-w)} \leq c_p < 1$ Now suppose that $0 \leq w < \frac{1}{3}$ and $1 - \frac{2}{3(1-w)} \leq c_p < 1$. Going back to the original inequality we want to show, to show that $\frac{d}{v_{nr}}\Pi_{II}(v_{nr})|_{v_{nr}=w+(1-w)\frac{1+c_p}{2}} < 0$ we need to show that

$$\begin{aligned} \pi\alpha(32c_p(w+c_p-wc_p)-(1+3w(-1+c_p)-3c_p)(1+w+c_p-wc_p)^2\pi\alpha) < \\ \sqrt{\pi\alpha\,(16c_p+(1+w+c_p-wc_p)^2\pi\alpha)}\Big(8w(1-c_p)+8c_p-\pi\alpha+ \\ (-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha\Big). \end{aligned} \tag{A.60}$$

When $0 \leq w < \frac{1}{3}$, $1 - \frac{2}{3(1-w)} \leq c_p \leq 1$, and $\pi\alpha > 0$, then $32c_p(w+c_p-wc_p)-(1+ 3w(-1+c_p)-3c_p)(1+w+c_p-wc_p)^2\pi\alpha > 0$ and $8w(1-c_p)+8c_p-\pi\alpha+ (-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha > 0$. Then similar to Subcase 1, both the left-hand side and right-hand side of the inequality are positive. We isolate the radicand and square both sides. Omitting the algebra, we again have the inequality (A.59).

When $c_p > 1 - \frac{2}{3(1-w)}$, the coefficient on the quadratic α term of (A.59) is negative, and the same argument as Subcase 1 applies to show that the inequality holds for all $\alpha > 0$ when $\pi > 0$, $0 \le w < \frac{1}{3}$, and $1 - \frac{2}{3(1-w)} \le c_p \le 1$.

On the other hand, if $c_p = 1 - \frac{2}{3(1-w)}$, then (A.58) reduces to $\pi\alpha(1-3w) - (1-w)\sqrt{\frac{\pi\alpha(3+\pi\alpha-w(9+\pi\alpha))}{1-w}}$, which holds for $\pi\alpha > 0$ and $0 \le w < \frac{1}{3}$. □

Subcase 2b: $0 \le w < \frac{1}{3}$ and $0 < c_p < 1 - \frac{2}{3(1-w)}$. Lastly, consider when $0 \le w < \frac{1}{3}$ and $0 < c_p < 1 - \frac{2}{3(1-w)}$.

Firstly, if $32c_p(w + c_p - wc_p) - (1 + 3w(-1 + c_p) - 3c_p)(1 + w + c_p - wc_p)^2\pi\alpha \ge 0$, then $8w(1 - c_p) + 8c_p - \pi\alpha + (-2 + 3w(-1 + c_p) - 3c_p)(w(-1 + c_p) - c_p)\pi\alpha > 0$ holds as well, for any $\pi > 0$, $\alpha > 0$, $w \in [0, 1]$, and $c_p \in (0, 1)$. In that case, again the inequality (A.58) would reduce to (A.59), and the same argument from Subcase 1 would apply to show that (A.58) holds.

On the other hand, consider if $32c_p(w + c_p - wc_p) - (1 + 3w(-1 + c_p) - 3c_p)(1 + w + c_p - wc_p)^2\pi\alpha < 0$. If $8w(1 - c_p) + 8c_p - \pi\alpha + (-2 + 3w(-1 + c_p) - 3c_p)(w(-1 + c_p) - c_p)\pi\alpha \ge 0$, then (A.58) holds without further conditions since the left-hand side would be negative while the right-hand side would be non-negative.

However, if $8w(1-c_p)+8c_p-\pi\alpha+(-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha < 0$, then we can divide by it and isolate the radicand of (A.58). Squaring both sides and omitting the algebra, this is equivalent to

$$\begin{aligned}64c_p(w + c_p - wc_p)^2 < -(&4(w(-1 + c_p) - c_p)(w^3(-1 + c_p)^3 + (-3 + c_p)c_p(-1 + 3c_p)+\\ &w(-1 + c_p)(1 + c_p)(1 + 11c_p) - w^2(1 - c_p)^2(2 + 15c_p))\pi\alpha+\\ &w(1 + 3w(-1 + c_p) - 3c_p)(-1 + c_p)(1 + w + c_p - wc_p)^3(\pi\alpha)^2) \quad \text{(A.61)}\end{aligned}$$

Since $0 \le c_p < 1 - \frac{2}{3(1-w)}$, the coefficient on the quadratic α term in (A.61) is positive. So to prove the inequality for all α, it suffices to show that the minimum of this quadratic in α is larger than $64c_p(w + c_p - wc_p)^2$. Finding the minimizer of the quadratic in α and comparing it to the lower bound on α given by $8w(1-c_p)+8c_p-\pi\alpha+(-2+3w(-1+c_p)-3c_p)(w(-1+c_p)-c_p)\pi\alpha < 0$, we find that the quadratic is minimized at $\alpha = \frac{8(w(1-c_p)+c_p)}{(1+3w(-1+c_p)-3c_p)(1+w+c_p-wc_p)\pi}$. The right-hand side of (A.61) evaluated at this α is indeed larger than $64c_p(w + c_p - wc_p)^2$ when $0 < c_p < 1 - \frac{2}{3(1-w)}$ and $0 \le w < \frac{1}{3}$. □

Then exhausting all sub-cases, it follows that (A.58) holds for all $w \in [0, 1]$. In particular, this means that $\frac{d}{v_{nr}}\Pi_{II}(v_{nr}) < 0$ for any $v_{nr} \ge \frac{1+c_p}{2}$. Therefore, $v_{nr} < \frac{1+c_p}{2}$ whenever $0 < v_{nr} < v_p < 1$ is induced in equilibrium. ■

Proof of Proposition 2: Under Condition Set Γ_3, for sufficiently low δ, the consumer market equilibrium structure that arises under optimal pricing is $0 < v_{nr} < v_r < 1$.

The vendor's price, provided in (A.43), is again given as

$$p^*_{III} = \frac{1}{9}\left(4 - \frac{1}{\pi\alpha} - \pi\alpha + \frac{\sqrt{1+\pi\alpha+(\pi\alpha)^3+(\pi\alpha)^4}}{\pi\alpha}\right). \tag{A.62}$$

Note that $\lim_{\pi\to 0} p^*_{III} = \frac{1}{2}$. Furthermore, $\frac{d}{d\pi}\left[p^*_{III}\right] < 0$ under the conditions of Γ_3. Therefore, $p^*_{III} < \frac{1}{2}$ for $\pi > 0$.

On the other hand, under Condition Set Γ_2, for sufficiently low δ, the consumer market equilibrium structure that arises under optimal pricing is $0 < v_r < 1$.

The vendor's price, provided in (A.49), is again given as

$$p^*_V = \Bigg(-1 - 2R\pi + 4\delta\pi\alpha - R^2\pi^2 - 2R\delta\alpha\pi^2 - (\delta\pi\alpha)^2 + (1 + R\pi + \delta\pi\alpha)\sqrt{1+\pi(2R-\delta\alpha+(R+\delta\alpha)^2\pi)}\Bigg)\Bigg(9\delta\pi\alpha\Bigg)^{-1}. \tag{A.63}$$

Then p^*_V has an asymptotic expression in δ given by

$$p^*_V = \frac{1}{2} - \frac{\pi\alpha}{8(1+R\pi)^2}\delta + \sum_{k=2}^{\infty} a_k\delta^k. \tag{A.64}$$

Then for sufficiently small δ, p^*_V is arbitrarily close to $\frac{1}{2}$. In particular, under the conditions of Γ_2, $\frac{d}{d\pi}\left[p^*_V\right] < 0$. Recall from Γ_2 that we have $\frac{\alpha-2R}{3R^2-4\alpha R+\alpha^2} < \pi < \underline{\pi} = \frac{c_p\alpha}{R^2-c_pR\alpha}$. Since an upper bound for π in Γ_2 is $\underline{\pi}$, it follows that a lower bound for p^*_V is $p^*_V|_{\pi=\underline{\pi}}$. This gives $p^*_V > \frac{1}{2} - \frac{c_p\alpha^2(R-c_p\alpha)\delta}{8R^3}$.

Since $p^*_{III} < \frac{1}{2}$ for $\pi > 0$ and $p^*_V > \frac{1}{2} - \frac{c_p\alpha^2(R-c_p\alpha)\delta}{8R^3}$, then for sufficiently small δ, there exists a intervals (π_1, π_2) and (π_3, π_4) where that $\pi_2 < \frac{\alpha-2R}{3R^2-4\alpha R+\alpha^2} < \pi_3$ such that for sufficiently small δ, the vendor's price (p^*_V) for any $\pi \in (\pi_3, \pi_4)$ is greater than his price (p^*_{III}) for any $\pi \in (\pi_1, \pi_2)$.

Lastly, under Condition Set Γ_1 (in particular, when $\pi > \pi_5$ for some $\pi_5 > \frac{c_p\alpha}{R^2-c_pR\alpha}$), the equilibrium market outcome is $0 < v_{nr} < v_r < v_p < 1$. The vendor's price, provided in (A.46), is given here again:

$$p^*_{IV} = \frac{R-c_p\alpha}{2R} + \frac{c_p\alpha^2(4c_p+3R\pi)\delta}{8R^3\pi} + \frac{c_p\alpha^3(16c_p^3\alpha - 4R^3\pi^2 + c_pR^2\pi(-18+5\pi\alpha) + 2c_p^2R(-8+9\pi\alpha))\delta^2}{16R^5(R-c_p\alpha)\pi^2} + \sum_{k=3}^{\infty} a_k\delta^k. \tag{A.65}$$

Taking the derivative with respect to π, we have $\frac{d}{d\pi}\left[p^*_{IV}\right] < 0$ for sufficiently small δ. Moreover, $\frac{R-c_p\alpha}{2R} < \frac{1}{2}$ under the conditions of Γ_1 so that for sufficiently small δ, $p^*_{IV} < p^*_V$. ■

Proof of Proposition 3: Under the conditions of Proposition 3, the equilibrium market outcome is either $0 < v_r < 1$ or $0 < v_{nr} < v_r < v_p < 1$, by Γ_1 and Γ_2. The boundary between these two cases in π is $\pi = \underline{\pi} = \frac{c_p\alpha}{R - c_p R\alpha}$.

The measures of interest are the size of the consumer segment willing to pay ransom and the expected total ransom paid. We examine each case separately, and then compare their measures at the boundary between the two cases.

For $0 < v_r < 1$, the size of the consumer segment willing to pay ransom in equilibrium is given as $r(\sigma^*) \triangleq 1 - v_r$. In this market structure, v_r was given by (A.48). We provide it again below.

$$v_r = \frac{-1 - R\pi + \delta\pi\alpha + \sqrt{4\delta\pi\alpha(p + R\pi) + (1 + R\pi - \delta\pi\alpha)^2}}{2\delta\pi\alpha}. \tag{A.66}$$

The vendor's price p_V^* for sufficiently small δ was given by (A.49).

Again, p_V^* has an asymptotic expression in δ given by

$$p_V^* = \frac{1}{2} - \frac{\pi\alpha}{8(1 + R\pi)^2}\delta + \sum_{k=2}^{\infty} a_k\delta^k. \tag{A.67}$$

Substituting this into the expression for v_r and simplifying $r(\sigma^*)$, we have the equilibrium size of the consumer segment willing to pay ransom is

$$r_V(\sigma^*) = \frac{1}{2(1 + R\pi)} + \sum_{k=1}^{\infty} a_k\delta^k. \tag{A.68}$$

Note that this is strictly decreasing in π for sufficiently small δ.

On the other hand, for $0 < v_{nr} < v_r < v_p < 1$, the size of the consumer segment willing to pay ransom in equilibrium is given as $r(\sigma^*) \triangleq v_p - v_r$. In this market structure, v_r and v_p were given by Case (IV) in Lemma A.1. In particular, $v_r = \frac{R}{\pi\alpha(1-\delta)}$ and $v_p = v_{nr} + \frac{v_{nr} - p}{\pi\alpha v_{nr}}$. The asymptotic expression for v_{nr} is given in (A.45). Substituting in the vendor's price, given in (A.46) and simplifying $r(\sigma^*)$, we have the equilibrium size of the consumer segment willing to pay ransom in this case is

$$r_{IV}(\sigma^*) = \frac{1}{2} - \frac{R}{\alpha} + \frac{c_p}{R\pi} + \sum_{k=1}^{\infty} a_k\delta^k. \tag{A.69}$$

Note that this is also strictly decreasing in π for sufficiently small δ.

To show that $r(\sigma^*)$ decreases for $\pi > \frac{\alpha - 2R}{3R^2 - 4\alpha R + \alpha^2}$, we need to show that there is a drop in the $r(\sigma^*)$ at the boundary $\pi = \underline{\pi}$ of these two cases. Note that $\lim_{\pi \to \underline{\pi}} r_V(\sigma^*) = \dfrac{R - c_p\alpha}{2R}$ while $\lim_{\pi \to \underline{\pi}} r_{IV}(\sigma^*) = \dfrac{1}{2} - c_p$. Given the conditions of the focal region, it follows that $\frac{R - c_p\alpha}{2R} > \frac{1}{2} - c_p$, completing the proof of the first statement in the proposition.

For the expected total ransom paid, that is given by $T(\sigma^*) \triangleq \pi u(\sigma^*) r(\sigma^*) R$, where $u(\sigma^*)$ is the size of the consumer segment willing to remain unpatched.

For $0 < v_r < 1$, this is given as

$$T_V(\sigma^*) = \frac{R\pi}{4(1+R\pi)^2} + \sum_{k=1}^{\infty} a_k \delta^k. \tag{A.70}$$

For $0 < v_{nr} < v_r < v_p < 1$, this is given as

$$T_{IV}(\sigma^*) = c_p \left(\frac{1}{2} - \frac{R}{\alpha} + \frac{c_p}{R\pi} \right) + \sum_{k=1}^{\infty} a_k \delta^k. \tag{A.71}$$

Under the conditions of Γ_2, we have that $\frac{d}{d\pi}\left[T_V(\sigma^*)\right] > 0$, while under the conditions of Γ_1, we have that $\frac{d}{d\pi}\left[T_{IV}(\sigma^*)\right] < 0$. This completes the proof. ■

Proof of Proposition 4: If $0 < v_{nr} < v_r < v_p < 1$ is induced in equilibrium, then from (A.47), the vendor's profit is given by

$$\Pi^*_{IV} = \frac{R - c_p\alpha}{4R} + \frac{c_p\alpha^2(2c_p + R\pi)\delta}{8R^3\pi} + \sum_{k=2}^{\infty} a_k \delta^k, \tag{A.72}$$

for some sequence of coefficients a_k.

Differentiating with respect to δ, we have that $\frac{d}{d\delta}\left(\Pi^*_{IV}\right) = \frac{c_p\alpha^2(2c_p+R\pi)}{8R^3\pi} + \sum_{k=1}^{\infty} b_k \delta^k$ for some sequence of coefficients b_k. It follows that for sufficiently small δ, $\frac{d}{d\delta}\left(\Pi^*_{IV}\right) > 0$.

The aggregate unpatched loss measure given as

$$UL \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,NR)\}} \pi\alpha u(\sigma^*) v dv + \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,R)\}} \pi u(\sigma^*)(R + \delta\alpha v) dv\,.$$

Under condition set Γ_1, the equilibrium outcome is given as $0 < v_{nr} < v_r < v_p < 1$. Consequently, aggregate unpatched losses are given as

$$UL^*_{IV} = \int_{v_{nr}}^{v_r} \pi\alpha(v_p - v_{nr}) v dv + \int_{v_r}^{v_p} \pi(v_p - v_{nr})(R + \delta\alpha v) dv\,. \tag{A.73}$$

Substituting (A.46) into (A.45), we can characterize the equilibrium v_{nr} threshold as

$$v_{nr}(p^*_{IV}) = \frac{1}{2} + \frac{c_p\alpha^2\delta}{8R(R - c_p\alpha)} + \sum_{k=2}^{\infty} a_k \delta^k. \tag{A.74}$$

Then substituting (A.46) and (A.74) into (A.24), we can characterize the equilibrium v_p threshold as

$$v_p(p_{IV}^*) = \frac{1}{2} + \frac{c_p}{R\pi} + \frac{c_p\alpha(8c_p^2\alpha + R^2\pi(-4+\pi\alpha) + 4c_pR(-2+\pi\alpha))\delta}{8R^3(R-c_p\alpha)\pi^2} + \sum_{k=2}^{\infty} a_k\delta^k. \quad \text{(A.75)}$$

Finally, the asymptotic expansion of $v_r = \frac{R}{\alpha(1-\delta)}$ is given by

$$v_r(p_{IV}^*) = \frac{R(1+\delta)}{\alpha} + \sum_{k=2}^{\infty} a_k\delta^k. \quad \text{(A.76)}$$

Substituting in (A.74), (A.76), and (A.75) into the above expression, the asymptotic characterization of the aggregate unpatched losses is given as

$$UL_{IV}^* = \frac{c_p(8c_p\alpha - (-2R+\alpha)^2\pi)}{8R\pi\alpha} + \frac{c_p\left(\frac{R(2R-\alpha)(4R^3-4c_pR^2\alpha+R\alpha^2-2c_p\alpha^3)}{\alpha(-R+c_p\alpha)} - \frac{24c_p^2\alpha}{\pi^2} + \frac{2c_p(4R^2-8R\alpha+\alpha^2)}{\pi}\right)}{16R^3}\delta + \sum_{k=2}^{\infty} a_k\delta^k. \quad \text{(A.77)}$$

Taking the derivative with respect to δ, we have

$$\frac{d}{d\delta}[UL_{IV}^*] = \frac{c_p\left(\frac{R(2R-\alpha)(4R^3-4c_pR^2\alpha+R\alpha^2-2c_p\alpha^3)}{\alpha(-R+c_p\alpha)} - \frac{24c_p^2\alpha}{\pi^2} + \frac{2c_p(4R^2-8R\alpha+\alpha^2)}{\pi}\right)}{16R^3} + \sum_{k=1}^{\infty} a_k\delta^k. \quad \text{(A.78)}$$

We will show that $\frac{c_p\left(\frac{R(2R-\alpha)(4R^3-4c_pR^2\alpha+R\alpha^2-2c_p\alpha^3)}{\alpha(-R+c_p\alpha)} - \frac{24c_p^2\alpha}{\pi^2} + \frac{2c_p(4R^2-8R\alpha+\alpha^2)}{\pi}\right)}{16R^3} < 0$ under Γ_1. Multiplying both sides by π^2, this is equivalent to $\frac{R(-2R+\alpha)(-4R^3+4c_pR^2\alpha-R\alpha^2+2c_p\alpha^3)\pi^2}{\alpha(-R+c_p\alpha)} + 2c_p(4R^2 - 8R\alpha+\alpha^2)\pi - 24c_p^2\alpha < 0$. This is a quadratic in π, and the coefficient of the second-order term is negative under the conditions of Γ_1. For this to hold, either there are two real roots and π is larger than the larger root of that quadratic or smaller than the smaller root of that quadratic, or there are not two real roots (in which case the inequality is always satisfied).

When the roots exist, the larger of the two roots is given by

$$\tilde{\pi}_1 = \Bigg(c_p\alpha((R-c_p\alpha)(4R^2-8R\alpha+\alpha^2)+ \Big(-(R-c_p\alpha)(176R^5-16(2+11c_p)R^4\alpha+8(-3+4c_p)R^3\alpha^2-8(1+3c_p)R^2\alpha^3+ (-1+32c_p)R\alpha^4+c_p\alpha^5)\Big)^{\frac{1}{2}}\Bigg)\Big(R(2R-\alpha)(4R^3-4c_pR^2\alpha+R\alpha^2-2c_p\alpha^3)\Big)^{-1} \quad \text{(A.79)}$$

Under Condition Set Γ_1, one of the conditions is $\pi > \frac{c_p\alpha}{R^2-c_pR\alpha}$. We will show that $\frac{c_p\alpha}{R^2-c_pR\alpha} > \tilde{\pi}_1$ so that $\pi > \frac{c_p\alpha}{R^2-c_pR\alpha}$ implies that $\pi > \tilde{\pi}_1$. This implies that $\frac{d}{d\delta}[UL^*_{IV}] < 0$ for sufficiently small δ under the conditions of Γ_1.

The inequality $\frac{c_p\alpha}{R^2-c_pR\alpha} > \tilde{\pi}$ is equivalent to

$$\Big(24(R-c_p\alpha)\Big)\Bigg((R-c_p\alpha)(4R^2-8R\alpha+\alpha^2)- \Big(-(R-c_p\alpha)(176R^5-16(2+11c_p)R^4\alpha+8(-3+4c_p)R^3\alpha^2- 8(1+3c_p)R^2\alpha^3+(-1+32c_p)R\alpha^4+c_p\alpha^5)\Big)^{\frac{1}{2}}\Bigg)^{-1} < \frac{1}{R^2-c_pR\alpha} \quad \text{(A.80)}$$

Under the conditions of Γ_1, denominator of the left-hand side is negative, the numerator is positive, and the right-hand side is positive. So then $\frac{c_p\alpha}{R^2-c_pR\alpha} > \tilde{\pi}_1$, which implies that $\frac{d}{d\delta}[UL^*_{IV}] < 0$ for sufficiently small δ under the conditions of Γ_1.

Similarly, denote consumer surplus as $CS \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)\in\{(B,NP,NR),(B,NP,R),(B,P)\}\}} U(v,\sigma)dv$. Under the condition set Γ_1, this becomes

$$CS^*_{IV} = \int_{v_{nr}}^{v_r} v - p^*_{IV} - \pi\alpha(v_p - v_{nr})vdv + \int_{v_r}^{v_p} v - p^*_{IV} - \pi(v_p - v_{nr})(R+\delta\alpha v)dv + \int_{v_p}^{1} v - p^*_{IV} - c_p dv\,. \quad \text{(A.81)}$$

Substituting in (A.46), (A.74), (A.76), and (A.75) into the above expression, the asymptotic characterization of consumer surplus is given as

$$CS_{IV}^{*} = \frac{1}{8}\left(1 + c_p\left(-8 + \frac{4R}{\alpha} + \frac{3\alpha}{R}\right)\right) + \frac{c_p(4c_p^2\alpha^2 + c_p\alpha(-4R^2 + 4R\alpha - 3\alpha^2)\pi + R(4R^3 - 2R^2\alpha + R\alpha^2 - 2\alpha^3)\pi^2)}{8R^3\alpha\pi^2}\delta + \sum_{k=2}^{\infty} a_k\delta^k. \tag{A.82}$$

Taking the derivative with respect to δ, we have

$$\frac{d}{d\delta}[CS_{IV}^{*}] = \frac{c_p(4c_p^2\alpha^2 + c_p\alpha(-4R^2 + 4R\alpha - 3\alpha^2)\pi + R(4R^3 - 2R^2\alpha + R\alpha^2 - 2\alpha^3)\pi^2)}{8R^3\alpha\pi^2} + \sum_{k=2}^{\infty} a_k\delta^k. \tag{A.83}$$

We will show that $\frac{c_p(4c_p^2\alpha^2+c_p\alpha(-4R^2+4R\alpha-3\alpha^2)\pi+R(4R^3-2R^2\alpha+R\alpha^2-2\alpha^3)\pi^2)}{8R^3\alpha\pi^2} < 0$ under the conditions of Γ_1. This is equivalent to $c_p(4c_p^2\alpha^2 + c_p\alpha(-4R^2 + 4R\alpha - 3\alpha^2)\pi + R(4R^3 - 2R^2\alpha + R\alpha^2 - 2\alpha^3)\pi^2) < 0$. This is a quadartic in π, with negative second-order term. Again, for this to hold, either there are two real roots and π is larger than the larger root of that quadratic or smaller than the smaller root of that quadratic, or there are not two real roots (in which case the inequality is always satisfied). Under the conditions of Γ_1, the larger of the two roots is given by

$$\tilde{\pi}_2 = \frac{8c_p\alpha}{4R^2 - 4R\alpha + 3\alpha^2 + \sqrt{-48R^4 + 24R^2\alpha^2 + 8R\alpha^3 + 9\alpha^4}}. \tag{A.84}$$

Under the conditions of Γ_1, $\frac{c_p\alpha}{R^2-c_pR\alpha} > \tilde{\pi}_2$, and since $\pi > \frac{c_p\alpha}{R^2-c_pR\alpha}$ is a condition of Γ_1, it follows that $\pi > \tilde{\pi}_2$ when Γ_1 holds. Therefore, $\frac{d}{d\delta}[CS_{IV}^{*}] < 0$ for sufficiently small δ under the conditions of Γ_1. ■

Proof of Proposition 5: From the consumer utility function (A.1), a consumer of valuation v prefers (B, NP, R) over (B, NP, NR) iff $v - p - \pi u(\sigma)(R + \delta\alpha v) \geq v - p - \pi u(\sigma)\alpha v$. This is equivalent to $v \geq \frac{R}{\alpha(1-\delta)}$. Consequently, if $\frac{R}{\alpha(1-\delta)} > 1$ (or $\delta > 1 - \frac{R}{\alpha}$), then no consumer would prefer (B, NP, R) over (B, NP, NR).

As (B, NP, R) is a strictly dominated option under this condition, consumers are left with (NB), (B, NP, NR), and (B, P) as incentive-compatible choices. Consequently, when $\delta > 1 - \frac{R}{\alpha}$, the consumer market equilibrium characterization no longer depends on R or δ, and the complete characterization of the consumer market equilibrium is given in Lemma A.2.

In particular, if $p < 1$ and $c_p + (-1 + c_p + p)\pi\alpha < c_p^2$ are satisfied, then the equilibrium outcome is $0 < v_{nr} < v_p < 1$. From Lemma A.4, we have $v_{nr} < \frac{1+c_p}{2}$ in equilibrium for $0 < v_{nr} < v_p < 1$ to arise under optimal pricing. This implies that, in this case, $v = \frac{1+c_p}{2}$ gets positive surplus from remaining unpatched. Expressing this in terms of the consumer's utility function, we have $\frac{1+c_p}{2} - p - \pi\alpha(v_p - v_{nr})\frac{1+c_p}{2} > 0$. This is equivalent to $p < \frac{1+c_p}{2} - \pi\alpha(v_p - v_{nr})\frac{1+c_p}{2}$. Then in equilibrium under optimal pricing, $p < \frac{1+c_p}{2}$.

Then $p < 1$ is satisfied under optimal pricing. Furthermore, a sufficient condition for $c_p+(-1+c_p+p)\pi\alpha < c_p^2$ to be satisfied under optimal pricing is $c_p+\left(-1 + c_p + \left(\frac{1+c_p}{2}\right)\right)\pi\alpha < c_p^2$ using the upper bound for the vendor's optimal price in the preceding paragraph. This is equivalent to $\pi\alpha > \frac{2(1-c_p)c_p}{1-3c_p}$. Hence, if $\pi\alpha > \frac{2(1-c_p)c_p}{1-3c_p}$ and $\delta > 1 - \frac{R}{\alpha}$, then $0 < v_{nr} < v_p < 1$ is the equilibrium outcome, and no measures of interest change in R or δ. ■

Proof of Proposition 6: First, we will show that under the conditions of Γ_2, whether $0 < v_{nr} < 1$ or $0 < v_{nr} < v_p < 1$ arises under the benchmark case, we will still have $p^*_{RW} > p^*_{BM}$. Then we will prove that when under the conditions of either Γ_1, the equilibrium outcome of the benchmark case is $0 < v_{nr} < v_p < 1$ and compare the price when ransomware is present to the price in that benchmark regime.

Under Γ_2, the equilibrium outcome is $0 < v_r < 1$ so that the vendor's profit function is given as $\Pi_V(p) = p(1-v_r(p))$, where $v_r(p)$ comes from (A.28). Using the first-order condition to solve for the vendor's price, we have

$$p^*_V = \Bigg(-1 - 2R\pi + 4\pi\alpha\delta - R^2\pi^2 - 2R\delta\alpha\pi^2 - (\delta\pi\alpha)^2 + (1 + R\pi + \delta\pi\alpha)\sqrt{1 + \pi(2R - \alpha\delta + (R + \delta\alpha)^2\pi)}\Bigg)\Bigg(9\delta\pi\alpha\Bigg)^{-1}. \quad \text{(A.85)}$$

Then p^*_V has an asymptotic expression given by

$$p^*_{V,RW} = \frac{1}{2} + \sum_{k=1}^{\infty} a_k\delta^k. \quad \text{(A.86)}$$

First, suppose that the equilibrium of the benchmark case was $0 < v_{nr} < 1$. Then the expression for v_{nr} given a price p when $0 < v_{nr} < 1$ is the outcome is given in (A.8). The vendor's profit function of this case is given as $\Pi_I(p) = p(1 - v_{nr}(p))$. Substituting in (A.8) and solving for the first-order condition, we have that $p^*_I = \frac{-1+\pi\alpha(4-\pi\alpha)+\sqrt{1+\pi\alpha+(\pi\alpha)^3+(\pi\alpha^4)}}{9\pi\alpha}$. From this, we have $\frac{d}{d\pi}p^*_I < 0$ whenever $\pi\alpha < 1$. However, $0 < v_{nr} < v_p < 1$ is induced in the benchmark case whenever $\pi\alpha \geq \frac{(2-3c_p)c_p}{\alpha(1-2c_p)}$. Therefore, if $0 < v_{nr} < 1$ is induced, then $\pi < \frac{(2-3c_p)c_p}{\alpha(1-2c_p)}$. However, we have $\frac{(2-3c_p)c_p}{\alpha(1-2c_p)} < 1$ for $0 < c_p < \frac{1}{3}$, which means that $\frac{(2-3c_p)c_p}{\alpha(1-2c_p)} < 1$ under the conditions of Γ_2. Therefore, the derivative of the benchmark price with respect to π is negative under Γ_2 when $0 < v_{nr} < 1$ is induced. Noting that $p^*_I|_{\pi=\frac{2c_p}{\alpha}} < \frac{1}{2}$ from $0 < c_p < \frac{1}{3}$, it follows that $p^*_I|_{\pi=\frac{2c_p}{\alpha}} < p^*_{V,RW}$ for sufficiently small δ. Moreover, since the benchmark price is decreasing in π, we have $p^*_{I,BM} < p^*_{V,RW}$ for any $\pi > \max(\frac{\alpha-2R}{3R^2-4R\alpha+\alpha^2}, \frac{2c_p}{\alpha})$ whenever $0 < v_r < 1$ is induced in ransomware case and $0 < v_{nr} < 1$ is induced in the benchmark case.

On the other hand, if the equilibrium of the benchmark case was $0 < v_{nr} < v_p < 1$, then from (A.37), an expression of the vendor's price as a function of v_{nr} when $0 < v_{nr} < v_p < 1$ is induced in equilibrium is given by

$$p^*_{II,BM}(v_{nr}) = \frac{1}{2}v_{nr}\left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)}\right). \tag{A.87}$$

From Lemmas A.3 and A.4, we have that $\frac{1}{2} < v_{nr} < \frac{1+c_p}{2}$ when $0 < v_{nr} < v_p < 1$ is the equilibrium outcome. We will show that (A.87) is increasing in v_{nr} and then show that when evaluated at $v_{nr} = \frac{1+c_p}{2}$, the expression is lower than (A.86) for sufficiently low δ.

The expression $\frac{d}{dv_{nr}}p^*_{II,BM}(v_{nr}) > 0$ is equivalent to $2 + v_{nr}\pi\alpha\left(2 - v_{nr}\sqrt{\frac{\pi\alpha}{4c_p+v_{nr}^2\pi\alpha}}\right) > \sqrt{\pi\alpha(4c_p + v_{nr}^2\pi\alpha)}$. This simplifies to $(1 + v_{nr}\pi\alpha)\sqrt{4c_p + v_{nr}^2\pi\alpha} > \sqrt{\pi\alpha}(2c_p + v_{nr}^2\pi\alpha)$. This is true when $v_{nr} > \frac{1}{2}$, which holds by Lemma A.3. Hence, (A.87) is increasing in v_{nr}.

Now, (A.87) evaluated at $v_{nr} = \frac{1+c_p}{2}$ is given by

$$p^*_{II,BM}\left(\frac{1+c_p}{2}\right) = \frac{1}{4}(1+c_p)\left(2 + \frac{1}{2}(1+c_p)\pi\alpha - \sqrt{\pi\alpha\left(4c_p + \frac{1}{4}(1+c_p)^2\pi\alpha\right)}\right). \tag{A.88}$$

The inequality $p^*_{II,BM}\left(\frac{1+c_p}{2}\right) < \frac{1}{2}$ is equivalent to $\pi > \frac{2c_p}{\alpha(1+c_p)^2}$. We will show that a lower bound on π (when π is in either Γ_1 or Γ_2) is larger than $\frac{2c_p}{\alpha(1+c_p)^2}$.

The lower bound on π for Γ_2 is $\frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}$. First, note that $\frac{d}{R}\left[\frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}\right] > 0$, for $R > 0$ and $\alpha > 0$. So the lower bound on π decreases as R decreases. The lowest R can be to remain in Γ_1 or Γ_2 is $R = \frac{\alpha}{2-c_p}$. At this value of R, the lowest π can be is $\left(\frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}\right)|_{R=\frac{\alpha}{2-c_p}} = \frac{c_p(2-c_p)}{\alpha(1-c_p^2)}$. This value of π satisfies $\pi > \frac{2c_p}{\alpha(1+c_p)^2}$, so all π in either Γ_1 or Γ_2 do too. Combined with the conditions of Γ_2, this proves that there exists a bound $\tilde{\delta}$ and $\pi_1 < \frac{c_p\alpha}{R^2-c_pR\alpha}$ such that when $0 \leq \delta < \tilde{\delta}$, when $\pi \in \left(\frac{\alpha-2R}{3R^2-4R\alpha+\alpha^2}, \pi_{10}\right)$, then $p^*_{RW} > p^*_{BM}$.

On the other hand, consider when $\pi \in \left(\frac{c_p\alpha}{R^2-c_pR\alpha}, 1\right)$. Then since this parameter set is in Γ_1, the equilibrium outcome under ransomware is $0 < v_{nr} < v_r < v_p < 1$. The vendor's price of this case is given in (A.46). To show that this is smaller than the vendor's price in the benchmark case for sufficiently low δ, since (A.87) is increasing in v_{nr}, it suffices to show that (A.87) evaluated at a lower bound of v_{nr} is greater than (A.46).

Note that from Lemma A.3, we have that $\pi\alpha > \frac{4c_p}{1-c_p^2}$. However, $\frac{4c_p}{1-c_p^2} > \frac{(2-3c_p)c_p}{(1-2c_p)}$ so that $0 < v_{nr} < v_p < 1$ is always induced in the benchmark case rather than $0 < v_{nr} < 1$ when Γ_1 holds. We compare (A.46) to $p^*_{II,BM}\left(\frac{1}{2}\right) = \frac{1}{8}\left(4 + \pi\alpha - \sqrt{\pi\alpha(16c_p + \pi\alpha)}\right)$, since $v_{nr} > \frac{1}{2}$ by Lemma A.3. The expression $\frac{1}{8}\left(4 + \pi\alpha - \sqrt{\pi\alpha(16c_p + \pi\alpha)}\right) > \frac{R-c_p\alpha}{2R}$ is equivalent to $2c_p\alpha + R(-2R + \alpha)\pi > 0$, which is one of the conditions in Condition Set Γ_{1X} for the optimal price of $0 < v_{nr} < v_r < v_p < 1$ to be an interior solution (given in the proof of Proposition 1). Combined with the conditions of Γ_1, this proves that there exists a bound $\tilde{\delta}$ such that when $0 \leq \delta < \tilde{\delta}$, when $\pi \in \left(\frac{c_p \alpha}{R^2 - c_p R \alpha}, 1\right)$, then $p^*_{RW} > p^*_{BM}$.

Lastly, we show that in the benchmark case, the vendor's price will always be decreasing in π, even when the market structure changes. First, suppose that π is low enough that the equilibrium market structure under optimal pricing is $0 < v_n < 1$. Then given a price p, v_n was given in (A.7).

The vendor's profit function is then $\Pi_I(p) = p(1 - v_n(p))$, and the vendor's price is given as

$$p^*_I = \frac{\sqrt{\alpha^4\pi^4 + \alpha^3\pi^3 + \alpha\pi + 1} + \alpha\pi(4 - \alpha\pi) - 1}{9\alpha\pi}. \tag{A.89}$$

Taking the derivative with respect to π and noting that $\pi\alpha < 1$ for this case to arise in equilibrium, we have that the vendor's price decreases in π.

When π is sufficiently large, then the equilibrium market structure may change to one in which high-valuation consumers patch. At this point in π, we'll show that the vendor's price cannot correspond to a strategic price jump. Suppose to the contrary that the vendor hikes the price at some $\tilde{\pi}$, and let $\pi_1 < \tilde{\pi} < \pi_2$ be such that the equilibrium market outcome of π_1 is $0 < v_n < 1$ and the equilibrium outcome of π_2 is $0 < v_n < v_p < 1$. Let π_1 and π_2 arbitrarily close, so that given $\epsilon > 0$, π_2 and π_1 are chosen so that $\frac{\pi_2}{\pi_1} < 1 + \epsilon$. Suppose that the vendor's price at π_2 is greater than his price at π_1, so that $p^*(\pi_1) < p^*(\pi_2)$. Given π, denote the consumer indifferent between buying and not buying as $v_n(\pi)$. The net utility of an unpatched consumer when $\pi = \pi_1$ is given as $U(v, NP) = v - p(\pi_1) - \pi_1\alpha(1 - v_n(\pi_1))v$. The net utility of a consumer who patches if $U(v, P) = v - p(\pi_1) - c_p$. Then for $v = 1$ to prefer to remain unpatched at π_1, it must be that $c_p \geq \pi_1\alpha(1 - v_n(\pi_1))$.

Conversely, at $\pi = \pi_2$, for $v = 1$ to strictly prefer to patch, it must be that $c_p < \pi_2\alpha(v_p(\pi_2) - v_n(\pi_2))$, where $v_p(\pi_2)$ is the type indifferent between patching and not patching. Combined with the above inequality, we have that $\pi_1(1 - v_n(\pi_1)) < \pi_2(v_p(\pi_2) - v_n(\pi_2))$. This is equivalent to $1 - \frac{\pi_2}{\pi_1}v_p(\pi_2) < v_n(\pi_1) - \frac{\pi_2}{\pi_1}v_n(\pi_2)$. Since $\frac{\pi_2}{\pi_1}$ is arbitrarily close to 1 and $1 - v_p(\pi_2) > 0$, it follows that $v_n(\pi_1) > v_n(\pi_2)$. This means that the type $v = v_n(\pi_1)$ remains a customer even when $\pi = \pi_2$.

However, if $v = v_n(\pi_1)$ remains a customer even when $\pi = \pi_2$, then her net utility must non-negative at $\pi = \pi_2$. Since her net utility was exactly 0 at $\pi = \pi_1$, we have that $v_n(\pi_1) - p(\pi_2) - \pi_2\alpha(v_p(\pi_2) - v_n(\pi_2))v_n(\pi_1) \geq v_n(\pi_1) - p(\pi_1) - \pi_1\alpha(1 - v_n(\pi_1))v_n(\pi_1)$. This is equivalent to $p(\pi_1) + \pi_1\alpha(1 - v_n(\pi_1))v_n(\pi_1) \geq p(\pi_2) + \pi_2\alpha(v_p(\pi_2) - v_n(\pi_2))v_n(\pi_1)$. Since $p(\pi_2) > p(\pi_1)$ by assumption, we have $\pi_1\alpha(1 - v_n(\pi_1))v_n(\pi_1) > \pi_2\alpha(v_p(\pi_2) - v_n(\pi_2))v_n(\pi_1)$. This is equivalent to $1 - \frac{\pi_2}{\pi_1}v_p(\pi_2) > v_n(\pi_1) - \frac{\pi_2}{\pi_1}v_n(\pi_2)$, a contradiction to the inequality in the previous paragraph. Therefore, in the benchmark case, a change in market structure cannot be associated with a strategic increase in price.

Lastly, suppose that the equilibrium market structure is $0 < v_n < v_p < 1$. We will show that under the assumptions of the focal region, the vendor's price remains decreasing in π.

Using (A.13) and employing asymptotic analysis, the vendor's price when $0 < v_n < v_p < 1$ is the induced market structure is given by:

$$p_{II}^* = \frac{1-c_p}{2} - \frac{2c_p^2(1-3c_p)}{(1+c_p)^3\pi\alpha} - \frac{16c_p^3(-3+c_p(8+c_p(-5+8c_p)))}{(1+c_p)^7(\pi\alpha)^2} + \sum_{k=3}^{\infty} c_k \left(\frac{1}{\alpha}\right)^k. \quad \text{(A.90)}$$

Note that $\alpha > \underline{\alpha} = \frac{(2-c_p)^2 c_p}{(1-c_p)^2}$ and $c_p \in (\frac{1}{2}(3-2\sqrt{2}), \frac{1}{2}(2-\sqrt{2}))$ in our focal region imply that $\alpha > 1 + \frac{1}{\sqrt{2}} \approx 1.71$. Then note that $\frac{d}{d\pi}\left[\frac{1-c_p}{2} + \frac{2c_p^2(-1+3c_p)}{(1+c_p)^3\pi\alpha} - \frac{16c_p^3(-3+c_p(8+c_p(-5+8c_p)))}{(1+c_p)^7(\pi\alpha)^2}\right] < 0$ for all $\pi \in [0,1]$ when $\alpha < \frac{16c_p(-3+8c_p-5c_p^2+8c_p^3)}{(1+c_p)^4(-1+3c_p)}$. This condition on α holds under the conditions of the focal region (since $\frac{16c_p(-3+8c_p-5c_p^2+8c_p^3)}{(1+c_p)^4(-1+3c_p)} > \bar{\alpha}$ under $\frac{1}{2}(3-2\sqrt{2}) < c_p < \frac{1}{2}(2-\sqrt{2})$), which implies that the vendor's price remains decreasing in π when the induced market outcome is $0 < v_n < v_p < 1$. ■

Proof of Proposition 7: First, note that the conditions $\underline{c_p} < c_p < \overline{c_p}$, $\underline{\alpha} < \alpha < \overline{\alpha}$, and $\underline{R} < R < \tilde{R}$ are common to both conditions sets Γ_1 and Γ_2. The only difference between Γ_1 and Γ_2 are the π conditions. In particular, when $\pi > \underline{\pi} = \frac{c_p\alpha}{R^2 - c_p R\alpha}$, then $0 < v_{nr} < v_r < v_p < 1$ arises in equilibrium while if $\underline{\underline{\pi}} < \pi \leq \underline{\pi}$ (where $\underline{\underline{\pi}} = \frac{-2R+\alpha}{3R^2-4R\alpha+\alpha^2}$), then $0 < v_r < 1$ arises.

Under Γ_2, the equilibrium outcome is $0 < v_r < 1$ so that the vendor's profit function is given as $\Pi_V(p) = p(1-v_r(p))$, where $v_r(p)$ comes from (A.28). Using the first-order condition to solve for the vendor's price, we have

$$p_V^* = \Bigg(-1 - 2R\pi + 4\pi\alpha\delta - R^2\pi^2 - 2R\delta\alpha\pi^2 - (\delta\pi\alpha)^2 + (1 + R\pi + \delta\pi\alpha)\sqrt{1 + \pi(2R - \alpha\delta + (R+\delta\alpha)^2\pi)}\Bigg)\Bigg(9\delta\pi\alpha\Bigg)^{-1}. \quad \text{(A.91)}$$

Then the equilibrium v_r has an asymptotic expression given by

$$v_r^* = \left(1 - \frac{1}{2+2R\pi}\right) + \sum_{k=1}^{\infty} a_k \delta^k. \quad \text{(A.92)}$$

The welfare function of this case is given by

$$SW \triangleq \int_{\mathcal{V}} \mathbb{1}_{\{\sigma^*(v)=(B,NP,R)\}} v - \pi u(\sigma^*)(R + \delta\alpha v) dv\,.$$

Consequently, welfare is given as

$$SW_V^* = \int_{v_r}^{1} v - \pi(1 - v_{nr})(R + \delta\alpha v) dv\,. \quad \text{(A.93)}$$

Substituting in (A.92) into the above expression, the asymptotic characterization of the welfare is given as

$$SW_V^* = \frac{3+2R\pi}{8(1+R\pi)^2} + \sum_{k=1}^{\infty} a_k \delta^k. \tag{A.94}$$

In particular, when $\pi = \underline{\pi} = \frac{c_p\alpha}{R^2 - c_p R\alpha}$, then $SW_V^* = \frac{(R-c_p\alpha)(3R-c_p\alpha)}{8R^2} + \sum_{k=1}^{\infty} a_k\delta^k$.

On the other hand, in the benchmark case, we will put bounds on the equilibrium welfare of $0 < v_{nr} < v_p < 1$. From (A.37), we have that an expression of the vendor's optimal price as a function of v_{nr} when $0 < v_{nr} < v_p < 1$ is induced in equilibrium is given by

$$p_{II}^* = \frac{1}{2} v_{nr} \left(2 + \pi\alpha v_{nr} - \sqrt{\pi\alpha(4c_p + \pi\alpha v_{nr}^2)} \right). \tag{A.95}$$

We also have that $v_p = \frac{c_p v_{nr}}{v_{nr} - p}$ from (A.14). The welfare of this case is given by

$$SW_{II} = \int_{v_{nr}}^{v_p} v - \pi\alpha(v_p - v_{nr}) v dv + \int_{v_p}^{1} v - c_p dv\,. \tag{A.96}$$

Substituting in (A.14) for v_p, we have that the welfare expression as a function of v_{nr} is given as

$$SW_{II}(v_{nr}) = \Bigg(- v_{nr}^3(\pi\alpha)^2 + c_p\sqrt{\pi\alpha(4c_p + v_{nr}^2\pi\alpha)} + \pi\alpha\Big(2 + c_p(-4 + v_{nr}) + v_{nr}^2\big(-2 + \sqrt{\pi\alpha(4c_p + v_{nr}^2\pi\alpha)}\big)\Big)\Bigg)\Big(4\pi\alpha\Big)^{-1}. \tag{A.97}$$

Using $v_{nr} > \frac{1}{2}$ from Lemma A.3, that (A.97) is strictly decreasing in v_{nr} for any $\pi > 0$, $\alpha > 0$, and $R > 0$ follows from the condition $c_p < \frac{1}{3}$. This condition holds under $c_p < \overline{c_p}$.

Since the welfare function strictly decreases in v_{nr}, it follows that the equilibrium welfare must be at least as much as the welfare function evaluated at an upper bound on v_{nr}. In particular, we use Lemma A.4 to have that $\frac{1+c_p}{2}$ is an upper bound on the equilibrium v_{nr}. Then the equilibrium welfare of $0 < v_{nr} < v_p < 1$ must be more than (A.97) evaluated at $v_{nr} = \frac{1+c_p}{2}$. Under the set of overlapping conditions of Γ_1 and Γ_2 stated in the proposition, $SW_{II}(\frac{1+c_p}{2})|_{\pi=\underline{\pi}} > SW_V^*|_{\pi=\underline{\pi}}$. Since the region of Γ_2 extends from $\pi = \underline{\underline{\pi}}$ to $\pi = \underline{\pi}$, it follows that for some small enough interval to the left of $\pi = \underline{\pi}$, the welfare of the benchmark case dominates the social welfare expression when ransomware is present in the threat landscape. Hence, there exists an $\epsilon > 0$ such that for sufficiently small δ, if $\frac{c_p\alpha}{R^2 - c_pR\alpha} - \epsilon < \pi < \frac{c_p\alpha}{R^2 - c_pR\alpha}$, then $SW_{RW} < SW_{BM}$.

Now consider when π is sufficiently small. Note that in the benchmark case, for $0 < v_{nr} < v_p < 1$ to hold, one of the conditions is that $c_p + (-1 + c_p + p)\pi\alpha < c_p^2$. For sufficiently small

π, this cannot hold since it becomes $c_p > 1$. Therefore, $0 < v_{nr} < 1$ arises in equilibrium for sufficiently small π in the benchmark case. The asymptotic expression for the welfare in this case is given by

$$SW_I = \frac{3}{8} - \frac{\pi\alpha}{4} + \sum_{k=1}^{\infty} a_k \pi^k. \tag{A.98}$$

At the same time, under the ransomware case, we can immediately rule out the following market structures from arising in equilibrium since at least one of the conditions in (A.1) would fail to hold in their respective cases for sufficiently low π: $0 < v_{nr} < v_r < v_p < 1$, $0 < v_r < v_p < 1$, $0 < v_{nr} < v_p < 1$. Then note that we can rule out $0 < v_{nr} < 1$ from the condition $R < \alpha$, which is a subcondition in Γ_1 and Γ_2. Lastly, $0 < v_r < 1$ fails to hold under sufficiently small π under the conditions $R > c_p\alpha$, $R < \frac{\alpha(1+c_p)}{2}$, and $R > \frac{\alpha}{2-c_p}$ (subconditions of Γ_1 and Γ_2).

Therefore, the equilibrium market outcome must be $0 < v_{nr} < v_r < 1$. For sufficiently small π, the equilibrium welfare is given as

$$SW_{III} = \frac{3}{8} + \frac{(R^2 - 2R\alpha)\pi}{4\alpha} + \sum_{k=1}^{\infty} a_k \pi^k. \tag{A.99}$$

Comparing against (A.98), we see that ransomware dominates the benchmark for sufficiently low π. Therefore, there exists a bound an $\epsilon > 0$ such that if $0 \leq \pi < \epsilon$, then $SW_{RW} \geq SW_{BM}$ for sufficiently low δ.

Lastly, consider the case of sufficiently high π. Under the conditions of Γ_1, the equilibrium market outcome is given as $0 < v_{nr} < v_r < v_p < 1$. Then the asymptotic expression of the equilibrium welfare of this case is given as

$$SW_{IV} = \frac{1}{8}\left(3 + c_p\left(-8 + \frac{4R}{\alpha} + \frac{\alpha}{R}\right)\right) + \sum_{k=1}^{\infty} a_k \delta^k. \tag{A.100}$$

On the other hand, the benchmark case would have $0 < v_{nr} < v_p < 1$ arising in equilibrium for high π. A lower bound on the equilibrium welfare is then given as $SW_{II}(\frac{1+c_p}{2})|_{\pi=1}$, where SW_{II} comes from (A.97). This becomes $\frac{1}{32}\Bigg(16+4(-7+c_p)c_p+4c_p\sqrt{(1+c_p)^2+\frac{16c_p}{\pi\alpha}} - (1+c_p)^3\pi\alpha + (1+c_p)^2\Bigg(-4+\sqrt{\pi\alpha(16c_p+(1+c_p)^2\pi\alpha)}\Bigg)\Bigg)$. Under the condition $16c_pR^2 + R\Bigg(4c_p\alpha + (1+c_p)^3\alpha^2 - \alpha\sqrt{\alpha(16c_p+(1+c_p)^2\alpha)} - c_p^2\alpha\sqrt{\alpha(16c_p+(1+c_p)^2\alpha)} - 2c_p\alpha \times \Bigg(2\sqrt{(1+c_p)^2+\frac{16c_p}{\alpha}} + \sqrt{\alpha(16c_p+(1+c_p)^2\alpha)}\Bigg)\Bigg)\Bigg) + 4c_p\alpha^2 > 0$, (A.100) dominates this expression for sufficiently low δ. Note that this is a quadratic expression in R, and under the conditions $c_p \in [\underline{c_p}, \overline{c_p}]$ and $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, the discriminant of the quadratic is negative so that this condition holds. Therefore, under the assumptions of the proposition, there exists a bound an $\epsilon > 0$ such that if $1 - \epsilon < \pi \leq 1$, then $SW_{RW} \geq SW_{BM}$ for sufficiently low δ. ■